A NEWS-UTILITY THEORY FOR INATTENTION AND DELEGATION IN PORTFOLIO CHOICE*

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Abstract

Recent evidence suggests that investors either are inattentive to their portfolios or undertake puzzling rebalancing efforts. This paper develops a life-cycle portfolio-choice model in which the investor experiences loss-averse utility over news and can choose whether or not to look up his portfolio. I obtain three main predictions. First, the investor prefers to ignore and not rebalance his portfolio most of the time to avoid fluctuations in news utility. Such fluctuations cause a first-order decrease in expected utility because the investor dislikes bad news more than he likes good news. Consequently, the investor has a first-order willingness to pay a portfolio manager who rebalances actively on his behalf. Second, when the investor does look up his portfolio himself, he rebalances it extensively to hasten or delay the realization of good or bad news, respectively. Third, the investor can reduce time-inconsistent overconsumption by being inattentive. Quantitatively, I structurally estimate the preference parameters by matching stock shares over the life cycle. My parameter estimates are in line with the literature, generate reasonable intervals of inattention, and simultaneously explain consumption and wealth accumulation over the life cycle.

JEL Codes: G02, G11, D03, D14, D91.

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1 Introduction

Standard finance theory says that investors should frequently rebalance their portfolios to realign their actual stock shares with their target shares, as stock prices frequently undergo large fluctuations. However, Bonaparte and Cooper (2009), Calvet et al. (2009a), Karlsson et al. (2009), Alvarez et al. (2012), and Brunnermeier and Nagel (2008) find that investors are inattentive and this inattention has been shown to matter in the aggregate by Chien et al. (forthcoming), Reis (2006), Gabaix and Laibson (2001), and Duffie and Sun (1990). Moreover, if investors are attentive, their behavior does not seem to reflect well-defined target shares. Hackethal et al. (2012) and Bergstresser et al. (2009) argue that investors overpay for delegated portfolio management, Calvet et al. (2009b) and Meng (2013) document that investors have a tendency to sell winning stocks (the disposition effect), and Feldman (2010) and Choi et al. (2009) show that investors mentally separate their accounts.

This paper offers an explanation for both inattention and demand for delegated portfolio management that simultaneously speaks to both the disposition effect and the mental-accounting phenomenon by assuming that the investor experiences utility over news or changes in expectations about consumption. Such news-utility preferences were developed by Koszegi and Rabin (2006, 2007, 2009) to discipline the insights of prospect theory, and they have since been shown to explain a broad range of micro evidence. The preferences’ central idea is that bad news hurts more than good news pleases, which makes fluctuations in news utility painful in expectation and provides a micro foundation for inattention. This micro foundation has many implications for behavior and welfare. Most importantly, if a news-utility investor has access to both a brokerage and a checking account, he will choose to ignore his portfolio in the brokerage account and instead fund his consumption out of the checking account most of the time. The investor will also have a first-order willingness to pay for a portfolio manager to rebalance his portfolio actively for him, which reduces its risk. Occasionally, however, the investor must rebalance himself to smooth his consumption; in this case, he engages in behavior reminiscent of the disposition effect and realization utility in the spirit of Barberis and Xiong (2012). Moreover, the investor’s desire to separate accounts, his consumption, and his self-control problems are reminiscent of mental accounting in the spirit of Thaler (1980). In addition to all this, I present the preferences’ implications for non-participation and stock shares in the presence of stochastic labor income to show that news utility also addresses pertinent questions in life-cycle portfolio theory.

I first explain the preferences in detail. The investor’s instantaneous utility in each period consists of two components. First, “consumption utility” is determined by the investor’s consumption as in the standard model. Second, “news utility” is determined by compar-
ing the investor’s updated expectations about consumption with his previous expectations. Specifically, the investor experiences “contemporaneous news utility” upon comparing his present consumption with his expectations about consumption. By making this comparison, he experiences a sensation of good or bad news over each previously expected consumption outcome, where bad news hurts him more than good news pleases him. The investor also experiences “prospective news utility,” by comparing his updated expectations about future consumption with his previous expectations. In so doing, he experiences news utility over what he has learned about future consumption.

To build intuition for my results in the dynamic model, I first highlight two fundamental implications of news utility for portfolio choice in a static framework. First, because bad news hurts more than good news pleases, even small risks that result in either good or bad news will cause a first-order decrease in expected utility. Consequently, the investor is first-order risk averse. Thus, he will not necessarily participate in the stock market, and when he does he will choose a lower portfolio share of stocks than under standard preferences. First-order risk aversion and non-participation are prevalent even in the presence of stochastic labor income.1 Second, first-order risk aversion implies that the investor can diversify over time, which means that his portfolio share is increasing in his investment horizon.2 The investor considers the accumulated stock-market outcome of a longer investment horizon to be less risky relative to its return because the expected first-order news disutility increases merely with the square root of his horizon whereas the return increases linearly in his horizon.

These two implications extend to a fully dynamic life-cycle model in which the investor chooses how much to consume and how much to invest in a risk-free asset and a risky asset. To allow for inattention, I modify the standard life-cycle model by assuming that the investor adjusts his portfolio via a brokerage account that he can choose to ignore and has a separate checking account to finance consumption in those inattentive periods.3 The news-utility investor prefers to be inattentive for some periods if a period’s length becomes sufficiently short. Looking up the portfolio means facing either good or bad news,

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1 The result about non-participation in the presence of background risk stands in contrast to earlier analyses, such as those of Barberis et al. (2006) and Koszegi and Rabin (2007, 2009), which I will explain in more detail. In any case, labor income makes stock-market risk more bearable as news utility is proportional to consumption utility, meaning that fluctuations in good and bad news hurt less on the flatter part of the concave utility curve.

2 In the dynamic model, time diversification implies that the investor chooses a lower portfolio share toward the end of life. A lower portfolio share later in life is advised by financial planners; as a rule of thumb, it should equal 100 minus the investor’s age. This advice is explained in the standard model via variation in the wealth-income ratio, variation in risk aversion due to changes in wealth, or mean reversion in stock prices. Empirically, participation and portfolio shares are hump-shaped over the life cycle, as shown in Section 6 and in Ameriks and Zeldes (2004).

3 Beyond allowing for inattention, this model relaxes the degree of freedom associated with calibrating a period’s length; this would otherwise crucially affect the results due to the possibility for time diversification.
which causes a first-order decrease in expected utility as explained above. However, not looking up the portfolio means not being able to smooth consumption perfectly. However, imperfect consumption smoothing has only a second-order effect on expected utility because the investor deviates from an initially optimal path. Thus, the investor optimally decides to be inattentive for a first-order period of time. Unfortunately, in inattentive periods the investor cannot rebalance his portfolio. This increases the portfolio’s risk and generates a first-order willingness to pay a portfolio manager to rebalance it actively according to standard finance theory. Assuming plausible parameter values, these two effects have quantitative implications that corroborate the empirical evidence. In particular, the model matches the findings of Alvarez et al. (2012) and Bonaparte and Cooper (2009) that the typical investor rebalances his portfolio approximately once a year, and that of Chalmers and Reuter (2012) and French (2008), who find that the typical investor forgoes approximately 50 to 100 basis points of the market’s annual return for portfolio management.

Occasionally, the news-utility investor has to reconsider his consumption plans and rebalance his portfolio. I show that his optimal portfolio share decreases in the return realization, whereas the standard investor’s portfolio share remains constant; therefore, the news-utility investor rebalances extensively. Intuitively, in the event of a good return realization the investor will want to realize the good news about future consumption and so will sell the risky asset. In the event of a bad return realization, however, the investor has to come to terms with bad news about future consumption. He will keep the bad news uncertain by increasing his portfolio share, which allows him to delay its realization until the next period, by which point his expectations will have adjusted. Such behavior is reminiscent of the empirical evidence on the disposition and break-even effects, though it originates from news utility about future consumption rather than from risk-lovingness in the loss domain. However, extensive rebalancing may imply that the investor holds on to rather than buys the risky asset after the market goes down. The reason is that the investor’s end-of-period asset holdings might still increase in the return realization due to a decrease in his consumption-wealth ratio. Intuitively, if an adverse return realizes, the investor consumes relatively more from his wealth to delay the decrease in consumption until his expectations have decreased.

Beyond portfolio choice, this model makes predictions about consumption, which are

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4The disposition effect (Odean (1998)) is an anomaly related to the tendency of investors to sell winners (stocks that have gone up in value) but keep losers (stocks that have gone down in value) to avoid the realization of losses. Odean (1998) argues in favor of the purchase price as a reference point, but Meng (2013) shows that expectations as a reference point seem to explain the data even better. The break-even effect refers to the observation that people become less risk averse in the loss domain. This effect is documented by Lee (2004) for professional poker players and Post et al. (2008) for Deal-or-No-Deal game show participants.
reminiscent of those implied by mental accounting.\textsuperscript{5} In my model, the investor’s two accounts finance different types of consumption: the brokerage account finances future consumption while the checking account finances current consumption. The accounts also exhibit different marginal propensities to consume: an unexpected windfall gain in the brokerage account is integrated with existing stock-market risk and partially consumed, while a windfall gain in the checking account is entirely consumed immediately.\textsuperscript{6} However, for expected gains, the investor has a higher marginal propensity to consume out of the brokerage account than out of the checking account because he overconsumes time inconsistently out of the brokerage account, but consumes efficiently out of the checking account. Thus, the two accounts help to exercise self control, which I explain next.\textsuperscript{7}

The investor may behave time inconsistently because he takes his expectations as given when he thinks about increasing his consumption today. Yesterday, however, he also took into account how such an increase would have increased his expectations. Thus today he is inclined to increase his consumption above expectations compared to the optimal committed plan that maximizes expected utility. This time inconsistency depends on news utility and therefore on uncertainty. Because inattentive consumption is deterministic and restricted by the amount of funds in the checking account, the investor consumes efficiently when he is inattentive. Intuitively, time-inconsistent overconsumption would result in good news about today’s consumption but bad news about tomorrow’s consumption: as the investor dislikes bad news more than he likes good news, he does not overconsume. As a result, the investor would like to commit to lowering his attentive consumption and portfolio share, which would result in looking up his portfolio less frequently, but also to rebalancing even more extensively. Exploring the investor’s commitment problem for both consumption and portfolio choice allows me to conclude that inattention and separate accounts result in a first-order increase in welfare and a decrease in overconsumption; these are potentially important results for normative household finance.

Even when people are deliberately inattentive, it seems unlikely that they receive no signals at all about what is happening in their brokerage accounts. Therefore, I extend the model so that the investor does receive signals about the value of his brokerage account, and after each decides whether he will remain inattentive. I first argue that the model’s implications are completely unaffected by signals with low information content: regardless

\textsuperscript{5}Mental accounting (Thaler (1980)) refers to the phenomenon that people mentally categorize financial assets as belonging to different accounts and treat these accounts as non-fungible to exercise self control.

\textsuperscript{6}This windfall result was first obtained by Koszegi and Rabin (2009).

\textsuperscript{7}This result may speak to the puzzle that people simultaneously borrow on their credit cards and hold liquid assets. The investor would happily pay additional interest to separate his wealth in order to not reconsider his consumption plans too frequently and overconsume less.
of the realization of such signals, the investor chooses not to look up his portfolio. In this case, he does not increase his consumption, which is restricted by the funds in the checking account, because he would otherwise have to consume less in the next period. Following this, I use simulations to outline the implications of signals with information content high enough to affect the investor’s attentiveness and consumption behavior. I find that adverse signals induce the investor to ignore his portfolio; he behaves according to the “Ostrich effect” (Karlsson et al. (2009)).

To evaluate the model quantitatively, I structurally estimate the preference parameters by matching the average empirical life-cycle profile of portfolio shares using household portfolio data from the Survey of Consumer Finances (SCF). In this data, I control for time and cohort effects using a technique that solves the identification problem associated with the joint presence of age, time, and cohort effects with minimal assumptions (Schulhofer-Wohl (2013)). My estimates are consistent with existing studies, generate reasonable attitudes towards small and large wealth bets, and generate reasonable intervals of inattention. The model’s quantitative predictions about consumption and wealth accumulation are also consistent with the empirical profiles inferred from the Consumer Expenditure Survey (CEX).

I first review the literature in Section 2. In Section 3, I explore news utility in a static portfolio setting to illustrate several fundamental results. I proceed to dynamic portfolio theory in Section 4. I outline the model environment, preferences, and solution in Sections 4.1 to 4.3. In Section 5, I extend my previous results about inattention and time diversification to the dynamic setting and outline several more subtle comparative statics about inattention (Section 5.1), explain the investor’s motives for rebalancing (Section 5.2), illustrate the model’s implications for commitment, welfare, delegation, and mental accounting (Section 5.3), and explore an extension to the model in which the investor receives signals about his portfolio (Section 5.4). In Section 6, I empirically assess the quantitative performance of the model. Finally, I conclude the paper and discuss future research in Section 7.

2 Comparison to the Literature


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8 This technique is of special importance in this context because the life-cycle profiles of participation and shares are highly dependent on the assumptions the age-time-cohort identification is based on, as made clear by Ameriks and Zeldes (2004). Age profiles for participation and portfolio shares turn out to be hump-shaped over the life cycle.
ment: Haliassos and Michaelides (2003) and Campanale et al. (2012) assume transaction costs; Gomes and Michaelides (2005) assume Epstein-Zin preferences, stock market entry costs, and heterogeneity in risk aversion; Lynch and Tan (2011) assume predictability of labor income growth at a business-cycle frequency and countercyclical variation in volatility; Gormley et al. (2010) and Ball (2008) assume disaster risk and participation costs; and Campanale (2009) assumes the existence of participation costs and an underdiversified portfolio. All of these papers focus on jointly explaining the empirical profiles of life-cycle consumption, participation, and portfolio shares.

Information aversion has recently been explored by Andries and Haddad (2013), Artstein-Avidan and Dillenberger (2011), and Dillenberger (2010) under the assumption of disappointment-aversion preferences, as formalized by Gul (1991). Several related papers study information and trading frictions in continuous-time portfolio-selection models: Abel et al. (2013) assume that the investor faces information and transaction costs when transferring assets between accounts; Huang and Liu (2007) and Moscarini (2004) explore costly information acquisition; and Ang et al. (fthc) assume illiquidity. “Rational inattention” also refers to a famous concept in macroeconomics introduced by Sims (2003), which postulates that people acquire and process information subject to a finite channel capacity. Luo (2010) explores how inattention affects consumption and investment in an infinite-horizon portfolio-choice model. A similar idea is pursued by Nieuwerburgh and Veldkamp (2013), who explore how precisely an investor will want to observe each signal about various of his assets’ payoffs under conditions where the total amount of precision with which he can observe signals is constrained. Moreover, Mondria (2010) and Peng and Xiong (2006) study attention allocation of individual investors among multiple assets. Among many others, Mankiw and Reis (2002), Gabaix and Laibson (2001), and Chien et al. (fthca) explore the aggregate implications of assuming that a fraction of investors are inattentive.

I combine these two literatures by exploring the implications of inattention for portfolio choice in a standard discrete-time framework, which simultaneously explains life-cycle consumption and wealth accumulation as well as participation and shares.\footnote{Hereby, I confirm the analysis in Pagel (2013), which shows how news utility generates a realistic hump-shaped life-cycle consumption profile in addition to other life-cycle consumption phenomena. Moreover, Pagel (2012) shows that news utility makes an additional step towards solving the equity premium puzzle by explaining asset prices and simultaneously generating reasonable attitudes over small and large wealth bets, an important point in favor of the prospect-theory asset-pricing literature that was first emphasized by Barberis and Huang (2008, 2009).}

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et al. (2006). Moreover, Engelberg and Parsons (2013) show that hospital admissions for mental conditions in California increase after stock market declines. Lab and field evidence for myopic loss aversion is provided by Gneezy and Potters (1997), Gneezy et al. (2003), Haigh and List (2005), and Fellner and Sutter (2009), while, in a similar vein, Anagol and Gamble (2013), Bellemare et al. (2005), Zimmermann (2013), and Falk and Zimmermann (2014) test whether subjects prefer information to be presented to them “clumped” or “piece-wise.” The prediction that the investor will engage in extensive rebalancing after he looks up his portfolio contradicts the intuitive idea that people sell stocks when the market is going down. The latter behavior would imply an increasing portfolio share, of the sort predicted by theories of herding behavior and habit formation. Habit formation, though, is tested in Brunnermeier and Nagel (2008), who find evidence in favor of a constant or slightly decreasing portfolio share.\textsuperscript{10} I briefly extend their analysis to provide suggestive evidence for a decreasing portfolio share, of the kind predicted by news utility, using household portfolio data from the Panel Study of Income Dynamics (PSID).

This paper analyzes a generally-applicable model of preferences that has been used in various contexts to explain experimental and other microeconomic evidence.\textsuperscript{11} The preferences’ explanatory power in other contexts is important because I put emphasis on the potentially normative question of how often people should look up and rebalance their portfolios. While most of the existing applications and evidence consider the static model of Koszegi and Rabin (2006, 2007), however, I incorporate the preferences into a fully dynamic and stochastic model. The inattention result has been anticipated by Koszegi and Rabin (2009) in a two-outcome model featuring consumption in the last period and signals in all prior periods. Several of my additional results extend and modify Koszegi and Rabin (2007), who analyze the preferences’ implications for attitudes towards risk in a static setting.

\textsuperscript{10}The tests run by Brunnermeier and Nagel (2008) are replicated in Calvet et al. (2009a). In the basic regression, Calvet et al. (2009a) confirm a slightly decreasing portfolio share. However, once the authors control for the realized return of the portfolio, the portfolio share becomes increasing. Calvet et al. (2009b) document that investors sell a greater number of stocks that have outperformed the market relative to the amount of stocks that have underperformed, which corresponds to the prediction of extensive rebalancing.

\textsuperscript{11}Heidhues and Koszegi (2008, 2014), Herweg and Mierendorff (2012), and Rosato (2012) explore the implications for consumer pricing, which are tested by Karle et al. (2011), Herweg et al. (2010) do so for principal-agent contracts, and Eisenhuth (2012) does so for mechanism design. An insufficient list of papers providing direct evidence for Koszegi and Rabin (2006, 2007) preferences is Sprenger (2010) on the implications of stochastic reference points, Abeler et al. (2012) on labor supply, Gill and Prowse (2012) on real-effort tournaments, Meng (2013) on the disposition effect, and Ericson and Fuster (2011) on the endowment effect (not confirmed by Heffetz and List (2011)). Suggestive evidence is provided by Crawford and Meng (2011), Pope and Schweitzer (2011), and Sydnor (2010). All of these papers consider the static preferences, but as the dynamic preferences of Koszegi and Rabin (2009) are a straightforward extension. Moreover, the notion that people are loss averse with respect to news about future consumption is indirectly supported by all experiments, which use monetary payoffs because these concern future consumption.
3 News Utility in Static Portfolio Theory

I start with static portfolio theory to introduce the preferences and quickly illustrate several important predictions in a simple framework. I first outline the news-utility implications for participation and portfolio shares and then turn to time diversification and inattention. To simplify the exposition, I assume that the agent picks a share $\alpha$ of his wealth $W$ to be invested in a risky asset with log return $\log(R) = r \sim N(\mu - \frac{\sigma^2}{2}, \sigma^2)$ as opposed to a risk-free asset with log return $\log(R^f) = r^f$ and that short sale and borrowing are prohibited, so $0 \leq \alpha \leq 1$. Thus, the agent’s consumption is given by $C = W(R^f + \alpha(R - R^f)) \sim F_C$. I also approximate the log portfolio return $\log(R^f + \alpha(R - R^f))$ by $r^f + \alpha(r - r^f) + \alpha(1 - \alpha)\frac{\sigma^2}{2}$, as suggested in Campbell and Viceira (2002), and denote a standard normal variable by $s \sim N(0,1)$.

3.1 Predictions about participation and shares

I now explain news-utility preferences in this static setting. Following Koszegi and Rabin (2006, 2007), I assume that the agent experiences consumption utility, which corresponds to the standard model of utility and is solely determined by consumption $c$, as well as news utility, which corresponds to the prospect-theory model of utility determined by consumption $c$ relative to the reference point $r$. The agent evaluates $c$ relative to $r$ using a piecewise linear value function $\mu(\cdot)$ with slope $\eta$ and a coefficient of loss aversion $\lambda$, i.e., $\mu(x) = \eta x$ for $x > 0$ and $\mu(x) = \eta \lambda x$ for $x \leq 0$. The parameter $\eta > 0$ weights the news-utility component relative to the consumption-utility component, and $\lambda > 1$ implies that bad news is weighted more heavily than good news. Koszegi and Rabin (2006, 2007) allow for a stochastic reference point and make the central assumption that the distribution of the reference point represents the agent’s fully probabilistic rational beliefs about consumption. In the static portfolio framework outlined above, then, expected utility is given by

$$EU = E[\log(C) + \eta \int_{-\infty}^{C} (\log(C) - \log(c))dF_C(c) + \eta \lambda \int_{C}^{\infty} (\log(C) - \log(c))dF_C(c)].$$ (1)

The first term in Equation (1) is simply consumption utility, which I assume to correspond to log utility that is, $u(c) = \log(c)$. The two additional terms in Equation (1) correspond respectively to good and bad news utility. First, the agent experiences a sensation of good news upon evaluating each possible outcome $C$ relative to all outcomes $c < C$ that would have had lower consumption weighted by their probabilities. Second, he experiences a sensation of bad news upon evaluating each possible outcome $C$ relative to all outcomes $c > C$ that would have had higher consumption weighted by their probabilities. In expectation, good
and bad news partly cancel, and the second and third terms in Equation (1) can be simplified to \( E[\eta(\lambda - 1) \int_C^\infty (\log(C) - \log(c))dF_C(c)] \) so that only the overweighted part of the bad news remains.

**Lemma 1.** The news-utility agent’s optimal portfolio share can be approximated by

\[
\alpha^* = \frac{\mu - r^f + \sigma E[\eta(\lambda - 1) \int_s^\infty (s - \tilde{s})dF_s(\tilde{s})]}{\sigma^2}
\]

if \( 0 \leq \alpha^* \leq 1 \) and \( \alpha^* = 0 \) if the expression (2) is negative or \( \alpha^* = 1 \) if the expression (2) is larger than one.

Proofs of this and the following lemmas and propositions can be found in Appendix D.

### 3.1.1 Samuelson’s colleague, time diversification, and inattention

In order to illustrate this static model’s implications for time, I define \( \mu \triangleq h\mu_0, \sigma \triangleq \sqrt{h}\sigma_0, \) and \( r^f \triangleq hr^f_0. \) The parameter \( h \) can be interpreted as the agent’s horizon: for example, if \( \mu_0 \) and \( \sigma_0 \) were originally calibrated so as to match the stock market’s distributional properties at a monthly frequency, \( h = 3 \) would imply a quarterly frequency.\(^{12}\) In the following proposition, I formalize that the news-utility investor prefers not to invest in the risky asset if his horizon is too short, that he can diversify over time, and that he gains from being inattentive.

**Proposition 1.** *(Horizon effects on portfolio choice)*

1. *(Samuelson’s colleague and time diversification)* There exists some \( h \) such that the news-utility agent’s portfolio share is zero for \( h < h, \) whereas the standard agent’s portfolio share is always positive. Moreover, the news-utility agent’s portfolio share is increasing in \( h, \) whereas the standard agent’s portfolio share is constant in \( h. \)

2. *(Inattention)* The news-utility agent’s normalized expected utility, i.e., \( \frac{EU}{h} \) with \( W = 1, \) is increasing in \( h, \) whereas the standard agent’s normalized expected utility is constant in \( h. \)

To illustrate this proposition more formally and explain the intuition, I plug the redefined terms into Equation (2). As can be easily seen, the news-utility agent’s portfolio share is positive if and only if \( h \) is high enough:

\[
\frac{\mu_0 - r^f_0}{\sigma_0} > -\frac{\sqrt{h}}{h} E[\eta(\lambda - 1) \int_s^\infty (s - \tilde{s})dF_s(\tilde{s})] > 0.
\]

\(^{12}\)Alternatively, \( h \) could be interpreted as the number of i.i.d. draws of an independent gamble of which the agent observes the overall outcome.
The integral represents the expected bad news of the investment and is always negative such that the news-utility agent requires a higher excess return before he will invest in the stock market if $\lambda > 1$ and $\eta > 0$ that is, he is first-order risk averse.\textsuperscript{13} First-order risk aversion implies that the expected good and bad news of the investment correspond to a first-order decrease in expected utility that is proportional to $\sigma_0$ and therefore $\sqrt{h}$ whereas the return of the investment is proportional to $\mu_0 - r_0^f$ and therefore $h$. Thus, the agent refuses to invest in the stock market if $h$ is low and $\sqrt{h}$ is high. Similarly, $\alpha^*$ is decreasing in $h$ as the expected return of the investment increases by $h$ while its expected sensation of bad news increases merely by $\sqrt{h}$. Thus, when $h$ increases, the investment’s expected sensation of bad news increases by less than its expected return, which makes the investment more attractive, and the news-utility agent can diversify over time. By contrast, the standard agent will invest some fraction of his wealth whenever $\mu > r_0^f$ and his portfolio share is independent of $h$, as the expected mean of the investment increases in $h$ proportional to its variance.

Figure 1 displays the static portfolio share of the news-utility and standard agents as a function of $h$ and the coefficient of loss aversion $\lambda$.

Figure 1: News-utility and standard agents’ portfolio shares as a function of the investment horizon $h$ and the coefficient of loss aversion $\lambda$.

\textsuperscript{13}The standard agent’s portfolio share is positive whenever $\mu > r^f$ because he is approximately risk neutral for small risks, i.e., for small risks his concave utility function becomes approximately linear. The investment’s risk becomes small if $\alpha$ is positive but small. A zero portfolio share for $\mu > r_0^f$ requires a kink in the utility function, which is introduced by news utility if $\eta > 0$ and $\lambda > 1$. An increase in $\sigma$ is then associated with a first-order decrease in expected utility and the portfolio share. More formally, the terms of the agent’s first-order condition that depend on $\sigma$, i.e., $-\alpha \sigma^2 + \sigma E[\eta(\lambda - 1) \int_{r^f}^{\infty} (s - \tilde{s}) dF_s(\tilde{s})]$, can be approximated by a second-order Taylor expansion around $\sigma = 0$, i.e., $(E[\eta(\lambda - 1) \int_{r^f}^{\infty} (s - \tilde{s}) dF_s(\tilde{s})]) \sigma + (-2\alpha) \sigma^2$. In this second-order approximation, the news-utility term is proportional to $\sigma$ while the standard agent’s term is proportional to $\sigma^2$; thus, the former is a first-order and the latter is a second-order effect of uncertainty on the portfolio share and expected utility.

\textsuperscript{14}The parameter values for the annual horizon are $\mu_0 = 8\%$, $r_0^f = 2\%$, and $\sigma_0 = 20\%$ and the preference
I now give some background on these two results, starting with the Samuelson’s colleague story. In Samuelson’s seminal paper, his colleague refuses to accept a 50/50 bet in which he would lose $100 or win $200 but is willing to accept 100 such bets. Samuelson (1963) showed that under standard expected-utility theory if one such bet was rejected at all wealth levels, then any number of them should also be rejected. The reason is that both the mean and variance of the sum of 100 bets increase by 100 over the single bet. The same logic applies to investors who believe in diversification over time that is, who believe that stock-market risk is decreasing in the investment’s horizon. Because the accumulated stock-market return over a given horizon is the sum of the individual outcomes in each time period, time diversification does not exist under standard assumptions. Investors’ portfolio shares should therefore be independent of their investment horizons.

To connect my model to the welfare benefits of inattention, I consider the implications of \( h \) for expected utility per unit of time or investment that is, normalized by \( h \) and for \( W = 1 \). The news-utility agent’s normalized expected utility,

\[
\frac{EU}{h} = r^f + \alpha (\mu - r^f) - \frac{\alpha^2 \sigma^2}{2} + \frac{\sqrt{h}}{h} \alpha \sigma E[\eta(\lambda - 1) \int_{\bar{s}}^{\infty} (s - \bar{s})dF_s(\bar{s})]
\]

is increasing in \( h \). Again, the investment’s expected return increases at a higher rate than the expected sensation of bad news. In contrast, the standard agent’s normalized expected utility is constant in \( h \). I now give some background to this result. Benartzi and Thaler (1995) explain the intuition about Samuelson’s colleague by formally showing that people who are loss averse and who myopically evaluate the outcome of each gamble will find an individual gamble inherently less attractive than the accumulated outcome of several gambles. Myopically loss averse investors thus gain from evaluating their portfolio at long rather than short horizons, and can diversify over time. Myopic loss aversion assumes that the accumulated outcome of the gamble is evaluated rather than each individual gamble to explain the behavior of Samuelson’s colleague. In other words, Samuelson’s colleague has to be inattentive to each individual gamble. Here, I show that the intuition generalizes to a setting in which the reference point is endogenous and stochastic. An increase in \( h \) implies that the agent integrates all risk: he ignores and does not experience news utility over each realization of the independent gambles \( h \) represents; thus, he gains from being inattentive.

parameters are \( \eta = 1 \) and \( \eta(\lambda - 1) = 1, \eta(\lambda - 1) = 1.5, \) or \( \eta(\lambda - 1) = 2 \). These are standard parameters in the prospect-theory literature, as I argue in Section 6.1.2. \( \eta(\lambda - 1) \approx 2 \) implies that the equivalent Kahneman and Tversky (1979) coefficient of loss aversion is around 2, because the news-utility agent experiences consumption and news utility, whereas classical prospect theory effectively consists of news utility only, and consumption utility works in favor of any small-scale gamble.
3.1.2 Labor income and wealth accumulation

Before moving on to dynamic portfolio theory, I make a short digression to illustrate the implications of labor income and wealth accumulation to show that my theoretical results do not depend on the absence of labor income. To do this, I simply assume that the agent will receive riskless labor income $\bar{Y} \geq 0$ and risky labor income $Y = e^y \sim \log - N(\mu_y, \sigma_y)$. His risky labor income is correlated with the risky return with covariance $Cov(r, y) = \sigma_{ry}$.

Lowercase letters denote logs.

Lemma 2. In the presence of risky labor income, the news-utility agent’s optimal portfolio share can be approximated by

$$\alpha^* = \frac{1}{\rho} \left[ \frac{\mu - r_f + \sigma E[\eta(\lambda - 1) \int_s^\infty (s - \tilde{s})dF_s(\tilde{s})]}{\sigma^2} - \frac{1 - \rho \sigma_{ry}}{\sigma^2} \right].$$

(3)

The derivation of $\alpha^*$ and the consumption function’s log-linearization parameter $\rho$ can be found in Appendix B.\textsuperscript{15} The portfolio share is potentially zero, decreasing in $\sigma$, $\eta$, and $\lambda$, increasing in riskless labor income, and decreasing in wealth. Riskless labor income simply transforms the portfolio share by $1 + \frac{\bar{Y}}{R_fW}$. This effect is particularly important for low wealth levels, which implies that portfolio shares are decreasing at the beginning of life in the standard model but is at odds with the empirical evidence.\textsuperscript{16} This transformation does not affect participation because the news-utility agent refuses to invest in the stock market at any wealth level, if the expected excess return is not high enough. Risky labor income tends to increase portfolio shares and participation, if it is imperfectly correlated, and it thus serves as a hedge against stock-market risk. In the news-utility model, however, additional riskless wealth that enters in an additive manner buffers stock-market risk. Risk generates fluctuations in news utility, which are proportional to consumption utility. Accordingly, these fluctuations hurt less on a high part of the concave utility curve. For the same reason the news-utility agent has a first-order precautionary savings motive, which increases his wealth from the first period on. Because labor income is increasing in the beginning of life, this consideration may cause both shares and participation to be increasing in the beginning of life.\textsuperscript{17} That portfolio shares can be increasing at the beginning of life when income growth is high for high levels of risk aversion has first been noted by Viceira (2001) and Cocco et al. (2005).

\textsuperscript{15} The derivation follows Campbell and Viceira (2002) among others.

\textsuperscript{16} Refer to Ameriks and Zeldes (2004) for an analysis of portfolio share and participation profiles or this paper’s analysis of SCF data in Section 6.

\textsuperscript{17} Refer to Fernandez-Villaverde and Krueger (2007) for an examination of hump-shaped income profiles or this paper’s analysis of SCF data in Section 6.
In this approximation, the presence of stochastic labor income does not affect the agent’s participation constraint and the agent’s first-order risk aversion is preserved. This result about first-order risk aversion in the presence of background risk does not depend on the approximation and can also be illustrated via the agent’s risk premium when stock market risk goes to zero, as done in Appendix B. This result stands in contrast to earlier analyses, such as those of Barberis et al. (2006) and Koszegi and Rabin (2007, 2009). Barberis et al. (2006) consider utility specifications under which the agent exhibits first-order risk aversion at a single point. Background risk takes the agent away from this point and he becomes second-order risk averse with respect to additional risk. However, the reference point is stochastic in the present model, so that the agent exhibits first-order risk aversion over the entire support of background risk. Koszegi and Rabin (2007, 2009) find that the agent becomes second-order risk averse in the limit if background risk is large and utility linear. However, labor income risk is not large relative to stock market risk in a life-cycle portfolio framework and the agent’s utility function is unlikely to be linear in a model that is calibrated to realistic labor income and stock-market risk at a monthly to annual horizons.

4 News Utility in Dynamic Portfolio Theory

The previous results about time diversification show that the model’s implications are highly dependent on the length of one time period; this remains a worrisome degree of freedom for the application of news utility. In order to reduce the importance of this assumption and to further elaborate on the implications of the model’s period length, I now introduce a life-cycle portfolio-choice model that deviates from other dynamic models in that it allows the agent to choose to either look up or decline to look up his portfolio. I start with the model environment and then explain the dynamic preferences of Koszegi and Rabin (2009); I then extend my previous results to the dynamic setting and further describe the agent’s preference for inattention, explain his motivation for rebalancing, and illustrate his time-inconsistency problems and its implications for consumption, inattention, and rebalancing. Finally, I consider an extension of the model to signals about the market in order to demonstrate that the baseline equilibrium is robust to the presence of signals and illustrate some more refined results about information acquisition.

4.1 The life-cycle model environment

The agent lives for \(T\) periods indexed by \(t \in \{1, ..., T\}\). In each period \(t\), the agent consumes \(C_t\) and may invest a share \(\alpha_t\) of his wealth \(W_t\) either in a risk-free investment with deter-
ministic return \( \log(R^f_t) = r^f_t \) or a risky investment with stochastic return \( \log(R_t) = r_t \sim N(\mu - \frac{\sigma^2}{2}, \sigma^2) \). I assume that short sale and borrowing are prohibited. In each period \( t \), the agent may be inattentive and choose not to observe the realization of the risky asset \( r_t \). If the agent invests a positive amount of wealth in the risky asset in period \( t - i \), but is inattentive in periods \( t - i + 1, ..., t - 1 \), he cannot observe the realization of his wealth \( W_{t-i+1}, ..., W_{t-1} \); therefore, his wealth for consumption in inattentive periods \( C'_{t-i+1}, ..., C'_{t-1} \) has to be stored in a risk-free checking account that pays interest \( R^d = e^{rd} \). Thus, his budget constraint for any period \( t \), before which he has observed the realization of his wealth in period \( t - i \), is given by

\[
W_t = (W_{t-i} - C_{t-i} - \sum_{k=1}^{i-1} (\frac{1}{R^d})^k C'_{t-i+k})((R^f_i)^i + \alpha_{t-i}(\prod_{k=1}^{i} R_{t-i+k} - (R^f_i)^i)).
\] (4)

In words, the agent’s wealth in period \( t \) corresponds to all the wealth he did not consume in period \( t - i \) minus the wealth which has to be stored in the checking account for consumption in periods \( t - i + 1, ..., t - 1 \) multiplied by the cumulative portfolio return assuming that all the wealth in the stock market stays in the stock market. All the variables that are indexed by period \( t \) realize in period \( t \). As preferences are defined over outcomes as well as beliefs, I explicitly define the agent’s probabilistic “beliefs” about each of the model’s period-\( t \) variables from the perspective of any prior period. Throughout the paper, I assume rational expectations such that the agent’s beliefs about any of the model’s variables agree with the objective probabilities determined by the economic environment.

**Definition 1.** Let \( I_t \) denote the agent’s information set in some period \( t \leq t + \tau \). Then, the agent’s probabilistic beliefs about any model variable, call it \( X_{t+\tau} \), conditional on period \( t \) information is denoted by \( F^I_{X_{t+\tau}}(x) = Pr(X_{t+\tau} < x | I_t) \).

### 4.2 The dynamic preferences

In the dynamic model of Koszegi and Rabin (2009), the utility function consists of consumption utility, “contemporaneous” news utility about current consumption, and “prospective” news utility about the entire stream of future consumption. Thus, total instantaneous utility

\[\text{15}\]

\[\text{16}\]
in any period $t$ is given by

$$U_t = u(C_t) + n(C_t, F_{C_t}^{t-1}) + \gamma \sum_{\tau=1}^{T-t} \beta^\tau n(F_{C_{t+\tau}}^{t,t-1}). \quad (5)$$

The first term on the left-hand side of Equation (5), $u(C_t)$, corresponds to consumption utility in period $t$. The second term, $n(C_t, F_{C_t}^{t-1})$, corresponds to news utility over contemporaneous consumption; here the agent compares his present consumption $C_t$ with his beliefs $F_{C_t}^{t-1}$ about present consumption. According to Definition 1, the agent’s beliefs $F_{C_t}^{t-1}$ correspond to the conditional distribution of consumption in period $t$ given the information available in period $t - 1$. Contemporaneous news utility is given by

$$n(C_t, F_{C_t}^{t-1}) = \eta \int_{-\infty}^{C_t} (u(C_t) - u(c))dF_{C_t}^{t-1}(c) + \eta \lambda \int_{C_t}^{\infty} (u(C_t) - u(c))dF_{C_t}^{t-1}(c). \quad (6)$$

The third term on the left-hand side of Equation (5), $\gamma \sum_{\tau=1}^{\infty} \beta^\tau n(F_{C_{t+\tau}}^{t,t-1})$, corresponds to prospective news utility experienced in period $t$ over the entire stream of future consumption. Prospective news utility about consumption in period $t + \tau$ depends on $F_{C_{t+\tau}}^{t-1}$, the beliefs the agent entered the period with, and on $F_{C_{t+\tau}}^{t}$, the agent’s updated beliefs about consumption in period $t + \tau$. The prior and updated beliefs about $C_{t+\tau}$, $F_{C_{t+\tau}}^{t-1}$, and $F_{C_{t+\tau}}^{t}$, are not independent distribution functions because future uncertainty $R_{t+1}, ..., R_{t+\tau}$ is contained in both. Thus there exists a joint distribution, which I denote by $F_{C_{t+\tau}}^{t,t-1}$.

$^{19}$Koszegi and Rabin (2006, 2007) allow for stochastic consumption, represented by the distribution function $F_c$, and a stochastic reference point, represented by the distribution function $F_r$. Then, the agent experiences news utility by evaluating each possible outcome relative to all other possible outcomes

$$n(c, F_r) = \int_{-\infty}^{\infty} \left( \eta \int_{-\infty}^{c} (u(c) - u(r))dF_r(r) + \eta \lambda \int_{c}^{\infty} (u(c) - u(r))dF_r(r) \right)dF_c(c). \quad (7)$$

I calculate prospective news utility $n(F_{C_{t+\tau}}^{t,t-1})$ by generalizing this “outcome-wise” comparison, Equation (7), to account for the potential dependence of $F_r$ and $F_c$, i.e.,

$$n(F_{C_{t+\tau}}^{t,t-1}) = \int_{-\infty}^{\infty} \left( \eta \int_{-\infty}^{c} (u(c) - u(r))dF_r(r) + \eta \lambda \int_{c}^{\infty} (u(c) - u(r))dF_r(r) \right)dF_{C_{t+\tau}}^{t,t-1}(c, r). \quad (8)$$

If $F_r$ and $F_c$ are independent, Equation (8) reduces to Equation (7). However, if $F_r$ and $F_c$ are non-independent, Equation (8) and Equation (7) yield different values. Suppose that $F_r$ and $F_c$ are perfectly correlated, as though no update in information occurs. Equation (7) would yield a negative value because the agent experiences news disutility over his previously expected uncertainty, which is unrealistic. In contrast, Equation (8) would yield zero because the agent considers the dependence of prior and updated beliefs, which captures future uncertainty, thereby separating uncertainty that has been realized from uncertainty that has not been realized. Thus I call this comparison the separated comparison. Koszegi and Rabin (2009) generalize the outcome-wise comparison slightly differently to a “percentile-wise” ordered comparison. The separated
Because the agent compares his newly formed beliefs with his prior beliefs, he experiences news utility about future consumption as follows:

\[
\mathbf{n}(\mathbf{F}_{C_{t+\tau}}) = \int_{-\infty}^{\infty} (\eta \int_{-\infty}^{c} (u(c) - u(r)) + \eta \lambda \int_{c}^{\infty} (u(c) - u(r))) d\mathbf{F}_{C_{t+\tau}}(c, r).
\]  

(9)

The agent exponentially discounts prospective news utility by \( \beta \in [0, 1] \). Moreover, he discounts prospective news utility relative to contemporaneous news utility by a factor \( \gamma \in [0, 1] \). Thus, he puts the weight \( \gamma / \beta^\tau < 1 \) on prospective news utility about \( t + \tau \) consumption.

### 4.3 The life-cycle model’s solution

In order to obtain analytical results, I assume log utility \( u(c) = \log(c) \) and approximate the log portfolio return \( \log(R_t^f + \alpha(R_t - R^f)) \) by \( r^f + \alpha(r_t - r^f) + \alpha(1 - \alpha)\frac{\tau^2}{T} \). The agent’s life-time utility in each period is

\[
u(C_t) + \mathbf{n}(C_t, F_{C_t}^{t-1}) + \gamma \sum_{\tau=1}^{T-t} \beta^\tau \mathbf{n}(F_{C_{t+\tau}}^{t-1}) + E_t[\sum_{\tau=1}^{T-t} \beta^\tau U_{t+\tau}],
\]

(10)

with \( \beta \in [0, 1], \eta \in (0, \infty), \lambda \in (1, \infty), \) and \( \gamma \in [0, 1] \). I define the model’s “monotone-personal” equilibrium solution concept in the spirit of the preferred-personal equilibrium solution concept as defined by Koszegi and Rabin (2009). In period \( t \), the agent has looked up his portfolio in periods \( t - i \) and \( t - i - j_0 \).

**Definition 2.** The attentive consumption function in any period \( t \) is admissible if it can be written as \( C_t = g(W_t, T - t, r_t, \ldots, r_{t-i+1}) \) and satisfies \( \frac{\partial \log(C_t)}{\partial \sum_{j=1}^{T} r_{t+i+j}} > 0 \). The inattentive consumption function in any period \( t \) is admissible if it can be written as \( C^{in}_t = g^{in}(W_{t-i} - C_{t-i}, T - t) \) and satisfies \( \frac{\partial \log(C^{in}_t)}{\partial \sum_{j=1}^{T} r_{t-i-j_0+j}} > 0 \). The portfolio function in any period \( t \) is admissible if it can be written as \( \alpha_t = g_\alpha(T - t, j_1, r_t, \ldots, r_{t-i+1}) \). \( \{C^{in}_t, C_t, \alpha_t\}_{t \in \{1, \ldots, T\}} \) is a monotone-personal equilibrium if, in all periods \( t \), the admissible consumption and portfolio functions \( C^{in}_t, C_t, \) and \( \alpha_t \) maximize (10) subject to (4) under the assumption that all future consumption and portfolio functions correspond to \( C^{in}_t, C_{t+\tau}, \) and \( \alpha_{t+\tau} \). In each period \( t \), the agent takes his prior beliefs about consumption \( \{F_{C_{t+\tau}}^{t-1}\}_{\tau=0}^{T-t} \) as given in the maximization problem.

The monotone-personal equilibrium solution can be obtained by simple backward induction. The first-order condition is derived under the premise that the agent enters period \( t \), and ordered comparisons are equivalent for contemporaneous news utility. However, for prospective news utility, they are qualitatively similar but quantitatively slightly different. As a linear operator, the separated comparison is more tractable. Moreover, it simplifies the equilibrium-finding process because it preserves the outcome-wise nature of contemporaneous news utility.
takes his beliefs as given, optimizes over consumption, and expects to behave like this in the future. Thus, the equilibrium is time consistent in the sense that beliefs map into correct behavior and vice versa. I now briefly state the equilibrium consumption and portfolio functions to convey a general idea of the model’s solution. The derivation is explained in detail in Appendix C.

Proposition 2. There exists a unique monotone-personal equilibrium if \( \sigma \geq \sigma^* \) for all \( t \in \{1, \ldots, T\} \) and \( \lambda > \frac{1}{\gamma} + \Delta \), with \( \Delta \) small. The agent’s optimal level of attentive consumption is \( C_t = W_t\rho_t \) with

\[
\rho_t = \frac{1}{1 + \sum_{\tau=1}^{T-t} \beta^\tau \frac{1 + \gamma (\eta F_t(r_t) \ldots F_{t_{t-\gamma}}) + \eta \lambda (1 - F_t(r_t) \ldots F_{t_{t_{t-\gamma}}}))}{1 + \eta F_t(r_t) \ldots F_{t_{t-\gamma}} + \eta \lambda (1 - F_t(r_t) \ldots F_{t_{t_{t-\gamma}}})}.
\]

(11)

If the agent is attentive, his optimal portfolio share is

\[
\alpha_t = \frac{\mu - r^f + \frac{1 + \gamma \sum_{i=1}^{T-t-j_1} \beta^i}{\sum_{i=1}^{T-t-j_1} \beta^i} \sqrt{\frac{\eta \lambda}{2}} \sigma E[\eta (\lambda - 1) \int_0^\infty (s \tilde{s}) dF(\tilde{s})]}{1 + \gamma (\eta F_t(r_t) \ldots F_{t_{t-\gamma}}) + \eta \lambda (1 - F_t(r_t) \ldots F_{t_{t-\gamma}})}
\]

(12)

if he plans to next look up his portfolio in period \( t + j_1 \) and has looked it up in periods \( t - i \) and \( t - i - j_0 \). Moreover, the agent’s optimal level of inattentive consumption is \( C_t^{\text{in}} = (W_t - C_{t-i})(R^d)^{\rho_t^{\text{in}}} \) with \( \rho_t^{\text{in}} \) for \( k = 1, \ldots, j_1 - 1 \) determined by the following recursion

\[
\rho_t^{\text{in}} = \frac{1}{1 + \beta^{j_1} \sum_{\tau=0}^{T-t-j_1} \beta^\tau} (1 - \sum_{k=1}^{i-1} \rho_{t-i+k}^{\text{in}} - \sum_{k=1}^{j_1-1} \rho_{t+k}^{\text{in}}).
\]

(13)

Here, \( 0 \leq \alpha_t \leq 1 \) and \( \alpha_t = 0 \) if the expression (12) is negative and \( \alpha_t = 1 \) if the expression (12) is larger than one, as I assume that short sale and borrowing are prohibited. Moreover, note that \( j_1 \) that is, the number of periods the agent remains inattentive after having looked up his brokerage account in period \( t \), depends on \( T - t \) and \( \alpha_t \) and is not determined in closed form. All of the following propositions are derived within this model environment under the first-order approximation for the portfolio return and they all hold in any monotone-personal equilibrium.

4.4 Comparison of the news-utility and standard policy functions

By contrast, the standard agent is attentive in every period \( t \), his optimal consumption function is \( C_t^s = W_t^s \frac{1}{1 + \sum_{\tau=1}^{T-t} \beta^\tau} \), and his portfolio share is \( \alpha^s = \frac{\mu - r^f}{\sigma^2} \). I now briefly highlight three observations about how the news-utility agent’s policy functions compare to those of the standard agent. First, the news-utility agent may overconsume relative to the standard
agent in attentive periods, as the news-utility agent’s consumption-wealth ratio $\rho_t$, as shown in Equation (11), is higher than that of the standard agent. The news-utility agent overconsumes if he cares more about contemporaneous than prospective news that is, $\gamma < 1$, because he is subject to a time-inconsistency problem that I explain in Section 5.3. Second, the news-utility agent does not overconsume in inattentive periods because he does not experience news utility over inattentive consumption and $\gamma$ is then irrelevant, as can be seen in Equation (13). The agent does not overconsume in inattentive periods because any good news he could get in present consumption would be outweighed by bad news in future consumption. Absent overconsumption, I can assume that the agent’s optimal level of inattentive consumption has been stored in the checking account previously. Third, the news-utility agent’s portfolio share, as given in Equation (12), is reminiscent of the static model’s portfolio share. However, in the dynamic model it depends on the agent’s horizon, $T - t$, the number of periods he will remain inattentive, $j_1$, and the history of returns, $F_t(r_t)\ldots F_t(r_{t-i+1})$. I now explain all these implications in greater detail. Hereafter, I assume that $\gamma < 1$; however, all the results that depend on this assumption hold for $\gamma = 1$ too, if instead of log utility I assume power utility with a relative coefficient of risk aversion greater than one.

5 Theoretical Predictions

I first extend the implications of the static model to the dynamic environment. I then explain the model’s predictions about inattention and rebalancing as well as the agent’s time inconsistency in more detail and finally consider signals about the market.

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20I do not have to consider equilibria in which the agent chooses a suboptimal portfolio share to affect the attentiveness of future selves. Such equilibria are irrelevant because the agent is not subject to a time inconsistency with respect to his plan of when to look up his portfolio so long as he does not overconsume time inconsistently in inattentive periods, which is ensured with the restriction $\lambda > \frac{1}{\gamma} + \Delta$. In inattentive periods, the agent does not experience news utility in equilibrium because no uncertainty resolves and he cannot fool himself. Therefore, he is not going to deviate from his inattentive consumption path in periods $t - i + 1$ to $t - 1$ so long as $u'(\rho_{t-i+k}) (1 + \eta) < \beta' u'(\rho_{t-i+k+j}) (1 + \gamma \eta \lambda)$ for $k = 1, \ldots i - 2$ and $j = 1, \ldots, i - k - 1$. As $u'(\rho_{t-i+k}) \approx u'(\rho_{t-i+k+j})$, this condition is roughly equivalent to $\lambda > \frac{1}{\gamma}$. Under this condition the model’s recursive solution is the same whether I assume that the agent looks up his portfolio when he is inclined to do so given a comparison of expected utilities or whether I assume that the agent ex ante precommits to looking up his portfolio at a certain date. The agent behaves time inconsistently only with respect to his attentive consumption and portfolio shares, as I will explain in Section 5.3.
5.1 Predictions about inattention

5.1.1 Samuelson’s colleague, time diversification, and inattention

To extend the model’s predictions for the story of Samuelson’s colleague, time diversification, and inattention to the dynamic environment, I define $\mu \triangleq h\mu_0$, $\sigma \triangleq \sqrt{h}\sigma_0$, $r^f \triangleq hr_0^f$, and $\beta \triangleq \beta_0^h$.

Corollary 1. (Horizon effects on portfolio choice) For $\beta \in (1 - \Delta, 1)$, with $\Delta$ small and assuming that the agent has to look up his portfolio every period.

1. (Samuelson’s colleague and time diversification) There exists some $h$ such that the news-utility agent’s portfolio share is zero for $h < h$, whereas the standard agent’s portfolio share is always positive. The news-utility agent’s portfolio share is increasing in $h$, whereas the standard agent’s portfolio share is constant in $h$. Finally, if $\gamma < 1$ then the news-utility agent’s portfolio share is increasing in his horizon $T - t$, whereas the standard agent’s portfolio share is constant in $T - t$.

2. (Inattention) The news-utility agent’s normalized expected utility, i.e., $\frac{E_{t-1}[U_h]}{h}$ with $E_{t-1}[\log(W_t \rho_t)] = 0$, is increasing in $h$, whereas the standard agent’s normalized expected utility is constant in $h$.

The intuitions presented in Section 3.1 carry over to the dynamic model directly. I now proceed to the main proposition in the dynamic model.

Proposition 3. For $T$ large, there exist some $\bar{h}$ and some $\underline{h}$ with $\bar{h} > \underline{h}$ such that (1) if $h > \bar{h}$ the news-utility agent will be attentive in all periods, and (2) if $h < \underline{h}$ the news-utility agent will be inattentive in at least one period. The standard agent, by contrast, will be attentive in all periods independently of $h$.

The basic intuition for this proposition is that the agent will look up his portfolio in every period if periods are very long say, ten years each. However, if periods are very short say, one day each, the agent will find it optimal to be inattentive for a positive number of them. In effect, the agent trades off the benefits of consumption smoothing and the costs of experiencing news utility. The benefits to be gained from consumption smoothing are proportional to the length of a period $h$ and are second order because the agent deviates from an initially optimal consumption path. The costs of experiencing news utility are proportional to $\sqrt{h}$ and are first order. Thus as $h$ becomes smaller the benefits of consumption smoothing decrease relative to the costs of news utility. Moreover, inattention has the additional benefit

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that the agent overconsumes less when inattentive.\textsuperscript{21} To ease understanding of the model’s implications, I will explain the agent’s decision-making problem in four periods in his end of life, periods $T$, $T - 1$, $T - 2$, and $T - 3$. Let me start with the agent’s portfolio share in period $T - 1$, assuming that he has looked up his portfolio in period $T - 2$:

$$\alpha_{T-1} = \frac{\mu_0 - r^f_0 + \sqrt{\frac{h}{n}} \frac{\sigma_0 E[\eta(\lambda - 1) \int_{\tilde{s}}^\infty (s - \tilde{s})dF(\tilde{s})]}{1 + \gamma (\eta F_0(r_{T-1}) + \eta \lambda (1 - F_0(r_{T-1})))}}{\sigma_0^2}.$$  

For now, I ignore the term containing $F_0(r_{T-1})$, which I will explain in Section 5.2, and focus on the terms that are familiar from the static model. As can be seen, there exists a lower bound for $h$ such that the agent’s portfolio share in period $T - 1$ is zero for any realization of $r_{T-1}$. That is, he behaves like Samuelson’s colleague. If I now assume that $h = \overline{h}$, would the agent look up his portfolio in period $T - 1$ or be inattentive? Abstracting from the difference in consumption-utility terms, he will look up his portfolio iff

$$\alpha_{T-2}(1 + \gamma \beta)\sigma E[\eta(\lambda - 1) \int_{\tilde{s}}^\infty (s - \tilde{s})dF(\tilde{s})] > \alpha_{T-2}\sqrt{2}\sigma E[\eta(\lambda - 1) \int_{\tilde{s}}^\infty (s - \tilde{s})dF(\tilde{s})].$$

Contemporaneous and prospective news utility today is proportional to $1 + \gamma \beta$, while news utility tomorrow over the realization of returns both today and tomorrow is proportional to $\beta \sqrt{2}$. Because $1 + \gamma \beta > \beta \sqrt{2}$ is a reasonable parameter combination and the agent consumes more if he looks up his portfolio, $E_{T-2}[\log(C_{T-1})] > \log(C_{T-1}^{up})$, as can be seen in Equations (11) and (13), I conclude that he is not unlikely to look up his portfolio. Let me simply suppose that he does so and move on to the optimal portfolio share in period $T - 2$, which differs from that of period $T - 1$ in that the expected sensation of bad news, $\sigma E[\eta(\lambda - 1) \int_{\tilde{s}}^\infty (s - \tilde{s})dF(\tilde{s})]$, is multiplied by $\frac{1 + \gamma \beta}{1 + \gamma \beta^2} < 1$. Because I stipulated that $h = \overline{h}$, the portfolio share in period $T - 2$ has to be positive for some realizations of $r_{T-2}$; thus, the agent chooses a higher portfolio share early in life because he can diversify over time. Now, again omitting differences in consumption utilities, the agent will find it optimal to look up his portfolio in period $T - 2$ iff

$$(\alpha_{T-3}(1 + \gamma (\beta + \beta^2)) + \beta \alpha_{T-2}(1 + \gamma \beta))\sigma E[\eta(\lambda - 1) \int_{\tilde{s}}^\infty (s - \tilde{s})dF(\tilde{s})] > \beta \alpha_{T-3}\sqrt{2}(1 + \gamma \beta)\sigma E[\eta(\lambda - 1) \int_{\tilde{s}}^\infty (s - \tilde{s})dF(\tilde{s})].$$

\textsuperscript{21} Thus, the avoiding of overconsumption and of news utility are two forces that drive inattention. Which of these forces dominates cannot be inferred from the proof of Proposition 3. However, for the parameter ranges I consider I confirm quantitatively that the avoidance of news utility is the more important force.
This consistency constraint is much less likely to hold than the previous one because $\alpha_{T-1}$ is positive whereas $\alpha_{T-2}$ is zero. Thus, the agent experiences news utility, which is painful in expectation, in periods $T-2$ and $T-1$ rather than in period $T-1$ alone. Being inattentive in period $T-2$ therefore causes a first-order increase in expected utility that is proportional to $\sigma = \sqrt{h}\sigma_0$. The decision not to smooth consumption perfectly in period $T-2$, however, causes a decrease in expected utility that is proportional to his consumption level and thus proportional to the portfolio return, which in turn is proportional to $\mu = h\mu_0$. Finally, let me suppose that the condition does not hold that is, that the agent remains inattentive in period $T-2$ and move on to his portfolio share in period $T-3$ assuming he has looked up his portfolio in period $T-4$. The basic difference of his portfolio share in period $T-3$ from that in period $T-1$ is that the expected sensation of bad news is multiplied by $\frac{1+\gamma\beta}{1+\beta} \sqrt{\frac{2}{2}} < 1$. Likewise, the difference from his portfolio share in $T-2$ is that the expected sensation of bad news is multiplied by $\frac{\sqrt{2}}{2} < 1$. Accordingly, the agent will choose a higher portfolio share if he foresees being inattentive in period $T-2$, because the expected return of two periods is $2\mu$ while the expected sensation of bad news increases by $\sqrt{2}$.

5.1.2 Comparative statics about inattention

The agent trades off the benefits of perfect consumption smoothing against the costs of experiencing more news utility. Moreover, his portfolio share is higher if $j_1$ is high that is, if he plans to be inattentive for a while. $j_1$, in turn, is determined by the agent’s decision on whether or not to look up his portfolio. But what considerations affect this decision? I now illustrate several comparative statics about the costs and benefits of inattention. Suppose the agent plans to look up his portfolio in period $t + j_1$ and has done so in period $t - i$. When he makes this decision, he considers the fact that he would experience news utility if he looked up his portfolio but also the fact that he would consume more. Thus, he compares the “benefit of delaying news utility” in period $t$, as given by

$$-\sqrt{i}\alpha_{t-i}(1 + \gamma \sum_{\tau=1}^{T-t} \beta^{\tau})\sigma E[\eta(\lambda - 1) \int_s^{\infty} (s - \tilde{s})dF(\tilde{s})],$$

with the “cost of less consumption utility” in period $t$, as given by $\log(\rho_t^{in}) - E[\log(\rho_t)]$. However, he also considers the fact that he will experience more news utility in period $t + j_1$. The “cost of delayed news utility” in period $t + j_1$ is given by

$$-(\sqrt{j_1}E[\alpha_t] - \alpha_t - i\sqrt{i + j_1})(1 + \gamma \sum_{\tau=1}^{T-t-j_1} \beta^{\tau})\sigma E[\eta(\lambda - 1) \int_s^{\infty} (s - \tilde{s})dF(\tilde{s})].$$
Thus, in terms of news utility, the “benefit of inattention” is the “benefit of delaying news utility” plus the “cost of delayed news utility.” I first formalize the implications in the following proposition, and then explain each in detail.

**Corollary 2.** At the beginning of life, if \( t \) is small, the expected benefit of inattention is positive. Toward the end of life, if \( t \) is large, the expected benefit of inattention is decreasing. Moreover, the benefit of inattention is decreasing in \( r_{t-i} + \ldots + r_{t-i-j_0+1} \).

At the beginning of life the optimal portfolio share has converged such that \( E[\alpha_{t-i}] = E[\alpha_t] \) and the expected benefit of inattention is necessarily positive. As \( \sqrt{i} + \sqrt{j_1} > \sqrt{i} + j_1 \) the agent prefers to ignore his portfolio in order to reduce his overall expected disutility from fluctuations in news. Toward the end of life the expected benefit of inattention is decreasing, as \( E[\alpha_{t-i}] > E[\alpha_t] \). The optimal portfolio share converges when the agent’s horizon becomes large; correspondingly, it decreases rapidly toward the end of life. Therefore, \( i \) and \( j_1 \) are small toward the end of life, which itself results in more frequent readjustments, because the benefit of inattention, which is proportional to \( \sqrt{i} + \sqrt{j_1} > \sqrt{j_1 + i} \), is decreasing. Thus, the very practice of looking up the portfolio more frequently results in a reduction of the portfolio share, simply because the agent cannot benefit from inattention as much. Quantitatively, at the beginning of life, the portfolio share is typically nonzero and the average interval of inattention is between eight and thirteen months depending on the parametrization. At the end of life, the agent is attentive every period if his portfolio share is zero, because \( R^f > R^d \), and the agent decides to exit the risky asset market. Finally, if the agent has experienced an adverse return realization in period \( t - i \) such that \( \alpha_{t-i} > E[\alpha_t] \), which I will explain in the next section, the benefit of inattention is reduced and he looks up his portfolio again earlier.

Figure 2 displays comparative statics of the average interval of inattention with respect to \( \lambda \) and \( \gamma \). As can be easily seen, the average interval of inattention is increasing in \( \lambda \) and \( \gamma \) as both make news utility more important relative to consumption utility.\(^2\)

\(^2\)I assume that one period equals one month and thus calibrate the parameter values as \( 12\mu = 7\% \), \( 12r^f = 2\% \), and \( \sqrt{12}\sigma_r = 20\% \), \( \frac{T}{12} = 60 \), \( \beta = 0.995 \approx 0.98 \), and \( \eta = 1 \). For inattentive intervals between eight and thirteen months the approximation to the portfolio share, which improves for lower frequencies, is reasonably accurate (for comparison, Viceira (2001) among others assume a yearly frequency using the approximation in a standard life-cycle portfolio-choice model).
Figure 2: News-utility inattentiveness comparative statics for $\lambda$ and $\gamma$.

Figure 3 displays the news-utility and standard agents portfolio shares at different points over their life cycles indicating how long the news-utility agent is planning to be inattentive.

Figure 3: News-utility and standard agents’ portfolio shares at different points over the life cycle with $\lambda = 2$ and $\lambda = 2.5$.

It can be seen that the news-utility agent’s portfolio share is not only increasing in his horizon but also decreasing in the return realization, which I explain next.

5.2 Predictions about rebalancing

Obviously, if the agent remains inattentive, he does not rebalance his portfolio. However, if he looks up his portfolio, he engages in extensive rebalancing or buy-low-sell-high investing because his portfolio share is decreasing in the return realization. But the agent does not necessarily buy the risky asset in bad times; rather, his end-of-period asset holdings might still increase in the market return more than a constant portfolio share would imply.
5.2.1 Extensive rebalancing in life-cycle portfolio theory

I refer to extensive rebalancing as a buy-low-sell-high investment strategy that is, the portfolio share is decreasing in the return realization rather than being constant.

Definition 3. The agent rebalances extensively if $\frac{\partial \alpha_r}{\partial (r_{t+1} + \ldots + r_{t-1})} < 0$.

Proposition 4. If he looks up his portfolio, the news-utility agent rebalances extensively. Moreover, iff $\gamma < 1$, the degree of extensive rebalancing is decreasing in the agent’s horizon.

The basic intuition behind extensive rebalancing is that upon a favorable return realization the agent wants to realize the good news about consumption and liquidates his risky asset holdings. By contrast, upon an adverse return realization the agent would prefer not to realize all the bad news associated with future consumption. He wants to keep the bad asset holdings. By contrast, upon an adverse return realization the agent would prefer not to realize all the bad news associated with future consumption. He wants to keep the bad

To illustrate this result, I now explain the derivation of the optimal portfolio share in period $T - 1$, assuming that the agent has looked up his portfolio in period $T - 2$. The first derivative of the agent’s continuation value is explained in the static portfolio choice application (Section 3.1) and as above is given by $\beta(\mu - r^f - \alpha_{T-1}\sigma^2 + E[\eta(\lambda - 1) \int_{r}^{\infty} (r_T - \tilde{r})dF_r(\tilde{r})])$. Moreover, the portfolio share affects prospective news utility, as $log(C_T) = log(W_{T-1} - C_{T-1}) + r^f + \alpha_{T-1}(r_T - r^f) + \alpha_{T-1}(1 - \alpha_{T-1})\sigma^2$. Note that, $W_{T-1} - C_{T-1}$ and $\alpha_{T-1}$ are stochastic from the perspectives of both periods $T - 2$ but deterministic from the perspective of period $T - 1$, whereas the realization of $r_T$ is stochastic from the perspective of period $T - 2$ and $T - 1$. Because the agent takes his prior beliefs $F^{T-2}_{W_{T-1} - C_{T-1}}$ and $F^{T-2}_{\alpha_{T-1}}$ as given and $log(C_T)$ is increasing in $r_{T-1}$, the first derivative of prospective news utility with respect to the portfolio share is

$$\frac{\partial \gamma n(F_{C_T}^{T-1,T-2})}{\partial \alpha_{T-1}} = \gamma \beta(\mu - r^f + \alpha_{T-1}\sigma^2)(\eta F_r(r_{T-1}) + \eta \lambda(1 - F_r(r_{T-1}))).$$

This term implies that $\alpha_{T-1}$ is decreasing in the realization of $r_{T-1}$. If $F_r(r_{T-1})$ is high, then the agent experiences relatively good news about future consumption and $\alpha_{T-1}$ is relatively low. On the other hand, if $F_r(r_{T-1})$ is low then the agent experiences relatively bad news about future consumption and $\alpha_{T-1}$ is relatively high.23

23Additionally, a decreasing portfolio share implies that the news-utility agent’s portfolio holdings are predictable on the basis of past shocks. The standard agent’s risky asset holdings at the end of period $t$ are given by $(W_t - C_t)\alpha_t = ((W_{t-1} - C_{t-1})R^p_t - C_t)\alpha_t$ and thus are linearly increasing in the standard agent’s portfolio return in period $t$. By contrast, the news-utility agent’s risky asset holdings at the end of period $t$,
5.2.2 End-of-period asset holdings in life-cycle portfolio theory

Extensive rebalancing, however, does not necessarily imply that the agent buys the risky asset in the event of a bad return realization. Rather, he might leave his risky asset holdings untouched or even decrease them. In fact, his end-of-period risky asset holdings, $W_t(1 - \rho_t)\alpha_t$, might increase more with the market return than a constant portfolio and consumption share, such as displayed by the standard agent, would imply.

Corollary 3. If $\gamma < \bar{\gamma}$, the news-utility agent’s end-of-period asset holdings increase with the market return by more than a constant portfolio share would imply

$$\frac{\partial \left( (R_f) + \alpha_{t-1}(R_{t-1} - R_f) \right)(1 - \rho_t)\alpha_t}{\partial (R_{t-1} - R_f)} > \alpha_{t-1}(1 - \rho_t)\alpha_t.$$

In any period $t$, the agent’s end-of-period risky asset holdings are given by $W_t(1 - \rho_t)\alpha_t$ while the beginning-of-period asset holdings are given by $W_{t-1}(1 - \rho_{t-1})\alpha_{t-1}R_t$. I have shown that $\alpha_t$ is decreasing in the return realization. But, $1 - \rho_t$ that is, one minus the agent’s consumption-wealth ratio, is increasing in the return realization, as can be easily seen in Equation (11). If $1 - \rho_t$ is increasing in the return realization, then optimal end-of-period asset holdings in the event of a favorable shock are relatively high. Intuitively, the consumption-wealth ratio $\rho_t$ is increasing in the return realization because in the event of an adverse shock the agent wishes to delay the reduction in consumption until his expectations have decreased.24 This increase may dominate the desire for extensive rebalancing if $\gamma$ is small. For illustration, suppose $\gamma = 0$, in which case $\alpha_t$ is constant but $1 - \rho_t$ is increasing in the return realization. In this case, the agent engages in behavior akin of herding.25

5.2.3 The net effect on the agent’s asset holdings

Now I want to assess the net effect of the two motives: does the news-utility agent move money into or out of his risky account in the event of favorable or adverse shocks? I start by considering the standard agent, whose change in risky asset holdings is proportional to $(R_f + \alpha^s(R_t - R_f))(1 - \rho^s_t)\alpha^s - \alpha^s R_t$. Both terms of this expression are negative whenever assuming he has looked up his portfolio in period $t - 1$, are given by $(W_t - C_t)\alpha_t = ((W_{t-1} - C_{t-1})R_f^t - C_t)\alpha_t$ and are thus increasing in the portfolio return. However, they are not increasing linearly, but less than linearly in the neighborhood of the return’s mean, as $\frac{\partial \alpha_t}{\partial r_t} < 0$. Moreover, the period $t + 1$ change in risky asset holdings is given by $(R_f + \alpha_t(R_{t+1} - R_f))(1 - \rho_{t+1})\alpha_{t+1} - \alpha_t$ and is thus predictable by the period’s $t$ return realization whereas this depends only on the period $t + 1$ return realization in the standard model. Therefore, news-utility risky asset holdings are more smooth than the standard agent’s risky asset holdings. 24 This result about delaying consumption adjustments is analyzed in Pagel (2013), as it brings about excess smoothness in consumption, and Pagel (2012), as it brings about predictability in stock returns. 25 In a general-equilibrium asset-pricing model, in which consumption is exogenous and returns are endogenous, the latter motive dominates and implies countercyclical expected returns as shown by Pagel (2013).
the portfolio share is not larger than one that is, when \( \alpha^s < 1 \), and the return realization is not too low, so that the standard agent will typically move money out of the risky account and the amount of the money transfer is monotonically increasing in the return realization.

The news-utility agent’s change in risky asset holdings is proportional to \((R_f^t + \alpha_{t-1}(R_t - R_f))(1 - \rho_t)\alpha_t - \alpha_t R_t\). Again, both terms are likely to be negative as \( \alpha_{t-1} < 1 \); however, as both \( \alpha_t \) and \( \rho_t \) are decreasing in the realization of \( R_t \) the overall response becomes ambiguous instead of uniformly decreasing as was the case with the standard agent. Figure 4 illustrates that the news-utility variation in \( \alpha_t \) and \( \rho_t \) can simultaneously lead to the change in his risky asset holdings being biased towards zero for each \( t \), as in the first scenario, and induce the news-utility agent to sell stocks when the market is going down because \( \gamma \) is low, as in the second scenario.\(^{26}\)

![Figure 4: News-utility and standard agents’ changes to risky asset holdings for each \( t \in \{1, ..., T\} \).](image)

### 5.3 Commitment, welfare, delegation, and mental accounting

The monotone-personal equilibrium is different from the one the agent would like to pre-commit to, and he is thus subject to a commitment or time-inconsistency problem. The agent behaves time inconsistently because he enjoys the pleasant surprise of increasing his consumption or portfolio share above expectations today, even though yesterday he also considered that such an increase in his consumption and portfolio share would have increased his expectations. Thus, today’s self thinks about today’s consumption and portfolio share in an inherently different way from yesterday’s self. Furthermore, today’s self wants to consume and enjoy the good news of potentially higher future consumption before his expectations

\(^{26}\)The parameter values for the annual horizon are \( \mu = 8\% \), \( r^f = 2\% \), and \( \sigma_r = 20\% \) and the preference parameters are \( \eta = 1 \), \( \lambda = 2 \), and \( \gamma = 0.8 \) versus \( \gamma = 0.2 \).
catch up. In the next proposition, I formalize that the agent would like to precommit to consuming less, investing less, which induces him to look up his portfolio less frequently, but also to rebalancing more extensively if he does look up his portfolio.

**Proposition 5. (Comparison to the monotone-precommitted equilibrium)**

1. The monotone-precommitted consumption share does not correspond to the monotone-personal consumption share, if the agent is attentive and \( \gamma < 1 \). In the monotone-precommitted equilibrium, the agent chooses a lower consumption share and the gap increases in good states, i.e., \( \rho_t > \rho_t^c \) and \( \frac{\partial (\rho_t - \rho_t^c)}{\partial (r_t + \ldots + r_{t+1})} > 0 \). If the agent is inattentive, he chooses the same consumption share, i.e., \( \rho_t^{in} > \rho_t^{cin} \).

2. The monotone-precommitted portfolio share does not correspond to the monotone-personal portfolio share. In the monotone-precommitted equilibrium, the agent chooses a lower portfolio share and the gap increases in good states, i.e., \( \alpha_t > \alpha_t^c \) and \( \frac{\partial (\alpha_t - \alpha_t^c)}{\partial (r_t + \ldots + r_{t+1})} > 0 \).

3. The cost of lowered consumption utility that arise from not looking up one’s portfolio are lower on the precommitted path.

4. Extensive rebalancing is more pronounced on the precommitted path.

I first explain the precommitted equilibrium in greater detail. The monotone-precommitted equilibrium maximizes expected utility and is derived under the premise that the agent can precommit to an optimal history-dependent consumption path for each possible future contingency, and that he thus jointly optimizes over consumption and beliefs. By contrast, the monotone-personal equilibrium is derived under the premise that the agent takes his beliefs as given, which is why he would deviate from the optimal precommitted path. I define the model’s “monotone-precommitted” equilibrium in the spirit of the choice-acclimating equilibrium concept in Koszegi and Rabin (2007) as follows.

**Definition 4.** \( \{C_t^{in}, C_t, \alpha_t\}_{t \in \{1, \ldots, T\}} \) is a monotone-precommitted equilibrium if, in all periods \( t \), the admissible consumption and portfolio functions \( C_t^{in}, C_t, \) and \( \alpha_t \) maximize (10) subject to (4) under the assumption that all future consumption and portfolio functions correspond to \( C_{t+\tau}^{in}, C_{t+\tau}, \) and \( \alpha_{t+\tau} \). In each period \( t \), the agent’s maximization problem determines both his fully probabilistic rational beliefs \( \{F_{C_{t+\tau}}^{-1}(r)\}_{\tau=0}^{T-t} \) and his consumption \( \{C_{t+\tau}^{in}, C_{t+\tau}\}_{\tau=0}^{T-t} \).

The monotone-precommitted equilibrium is derived in Appendix C.2. Suppose that the agent can precommit to an optimal consumption path for each possible future contingency. In his optimization problem, the agent’s marginal news utility is no longer composed solely of the sensation of increasing consumption in that contingency; additionally,
the agent considers that he will experience fewer sensations of good news and more of bad news in all other contingencies. Thus marginal news utility has a second component, 

\[-u'(C_t)(\eta(1 - F_{C_t}^{t-1}(C_t)) + \eta\lambda F_{C_t}^{t-1}(C_t))\]

which is negative so that the precommitted agent consumes and invests less than the non-precommitted agent. This can be easily seen in Equations (11) and (12). The additional negative component dominates if the consumption realization is above the median that is, \(F_{C_t}^{t-1}(C_t) > 0.5\). Thus, in the event of good return realizations, precommitted marginal news utility is negative. By contrast, non-precommitted marginal news utility is always positive because the agent enjoys the sensation of increasing consumption in any contingency. Therefore the additional negative component to marginal news utility implies that the precommitted agent does not overconsume even if \(\gamma < 1\). Moreover, the difference between the precommitted and non-precommitted consumption paths is smaller in the event of adverse return realizations, because increasing risky asset holdings is the optimal response even on the precommitted path. Thus, the degree of the agent’s time inconsistency is reference dependent, which also implies that the motive for extensive rebalancing is more pronounced on the precommitted path. However, during inattentive periods the agent does not overconsume so long as \(\gamma > \frac{1}{\lambda}\), as explained in Section 4.3. Therefore, the difference between inattentive and attentive consumption is smaller for the precommitted agent, which reduces the benefits to be gained from looking up his portfolio.

A simple calculation reveals the quantitative magnitude of the welfare implications of inattention. Suppose a period is one month long. If the news-utility agent has to look up his portfolio every period, his portfolio share will be zero. If he can be inattentive, however, his wealth can achieve a return of around four percent per year, which accumulates over time. Thus, he will be willing to give up a considerable share of either his initial wealth or the return to his wealth to separate his accounts.

The agent also has a first-order willingness to pay for a portfolio manager who rebalances actively. The simple reason for this is that his log portfolio return is given by 

\[\log((R_f)^i + \alpha_t - i(\prod_{k=1}^{\tau} R_{t-i+k} - (R_f)^i))\]

while his return under active rebalancing would be given by 

\[\log(\prod_{k=1}^{\tau} (R_f + \alpha_t - i(R_{t-i+k} - R_f)))\].

The variance of the former is strictly higher than the variance of the latter, and the news-utility agent cares about risk to a first-order extent. Quantitatively, this effect matches the empirical evidence provided by Chalmers and Reuter (2012) and French (2008), who find that the typical investor forgoes approximately 50 to 100 basis points of the market’s annual return for delegated portfolio management.\(^{27}\)

\(^{27}\)As a back-of-the-envelope calculation, suppose the agent’s portfolio share is 0.4 and he looks up his portfolio once per year in line with the quantitative exercise in Section 5.1.2. Monthly as opposed to yearly rebalancing will result in a reduction of risk given by 0.3% and an increase in the expected return of around 0.04%. Thus, the agent would be willing to give up \((-0.3E[\eta(\lambda - 1) \int_s^\infty (s-\tilde{s})dF(\tilde{s}))] - 0.04)\frac{1}{\pi\chi}\%\) of the annual stock-market return, which matches the empirical evidence for \(\eta = 1\) and \(\lambda \approx 2.6\).
My results about consumption and welfare relate to the idea of mental accounting (Thaler (1980)). In my model, the investor’s two accounts finance different types of consumption: the brokerage account finances future consumption while the checking account finances current consumption. The accounts also exhibit different marginal propensities to consume as can be seen from the consumption shares in Equation (11). This difference in consumption shares help the agent to exercise self control because he overconsumes time inconsistently out of the brokerage account but consumes efficiently out of the checking account. Moreover, the two accounts treat windfall gains differently: an unexpected windfall gain in the brokerage account is integrated with existing stock-market risk and partially consumed, while a windfall gain in the checking account is entirely consumed immediately.\(^{28}\)

5.4 Extension to signals about the market

Even when people are deliberately inattentive, it seems unrealistic to assume that they do not receive any news about what is happening in their brokerage account. Therefore, I extend the model so that the agent receives a signal about the value of his asset holdings in the brokerage account and then decides whether or not to remain inattentive in each period. In Section 5.4.1, I first argue that the equilibrium under consideration will be completely unaffected by the signal if its information content is low. In this case, the signals do not affect either the agent’s attentiveness or his inattentive consumption at all, though they do affect his attentive consumption and rebalancing as the latter two behaviors depend on how the agent’s portfolio compares to the signals he received. In Section 5.4.2, I then outline the implications of signals that do have high enough information content to affect the agent’s attentiveness and thus his consumption plans using simulations. Nevertheless, I argue that even in these cases the signal’s effect on consumption and attentiveness is modest at best.

5.4.1 Signals with low information content

In the following, I consider signals about the market that have low information content and argue that the agent’s attentiveness and thus consumption behavior are completely unaffected. The basic idea is that the potential good news to be gained from looking up the portfolio, even if the signal happens to be particularly favorable, is outweighed by the expected disutility from paying attention. If the presence of the signal does not affect the agent’s decision whether to look up his portfolio, it does not affect his consumption out of the checking account either. The agent does not want to consume more in the event of a favorable

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\(^{28}\)This result may speak to the puzzle that people simultaneously borrow on their credit cards and hold liquid assets. The investor would happily pay additional interest to separate his wealth in order to not reconsider his consumption plans too frequently and overconsume less.
signal because such an increase in current consumption would imply a decrease in future consumption if he does not plan to change the date when he will next look up his brokerage account. Since the bad news about the decrease in future consumption outweighs the good news of overconsumption, the agent sticks to his original consumption plan independently of the signal. Thus, the agent experiences news utility merely over his consumption in the future after he has looked up his brokerage account.

5.4.2 Signals with high information content and the Ostrich effect

In the following, I will outline what happens if the signal's information content is so high that it does affect the agent’s plan of when to look up his portfolio. More specifically, I will show that the agent’s willingness to look up his portfolio increases with the realization of the signal. This allows me to conjecture how the equilibrium looks, given my previous findings.

I assume that the agent has looked up his portfolio in period $t - i$ and receives a signal $\hat{r}_t = r_t + \epsilon_t$ with $\epsilon_t \sim N(0, \sigma^2_\epsilon)$ about $r_t$ in period $t$. In period $t$, the agent will look up his portfolio after receiving a particularly favorable signal, will not react to signals in some middle range, and will resist looking up his portfolio following particularly bad signals. If he ignores his portfolio, he will not consume time inconsistently because he does not experience news utility over inattentive consumption; he experiences that only over future consumption after having looked up the portfolio. Absent time-inconsistent overconsumption during inattentive periods, his previous selves have no reason to restrict the funds in the checking account. His previous selves might want to affect his later decision whether or not to look up his portfolio in period $t$. Although his previous selves cannot prevent his period $t$ self from looking up the portfolio they can force his period $t$ self to look up his portfolio namely, when his period $t$ self runs out of funds in the checking account. However, they would never want to make him look up his portfolio earlier than his period $t$ self already wants to, because the precommitted path involves looking up the portfolio fewer times than the non-precommitted path. Because there is no time inconsistency associated with inattentive consumption, I can assume that the previous selves would have stored sufficient funds in the checking account to allow the investor to remain inattentive longer, in the event of adverse signals, until he would look up his portfolio in accord with the precommitted path.

Now I show that the agent is more likely to look up his portfolio after a favorable realization of the signal, a behavior that has been termed the Ostrich effect. If the agent looks up his portfolio, he will experience news utility over the actual realization of his portfolio. If he ignores his portfolio, he will experience news utility over the signal. His expectation of contemporaneous and prospective news utility in period $t$ is assessed conditionally on the signal. In particular, the agent expects contemporaneous and prospective news from looking
up his return as follows:

\[ \alpha_{t-i}(1 + \gamma \sum_{\tau=1}^{T-t} \beta^\tau) \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \eta(\lambda - 1)(r - \tilde{r})dF_{r_{t+...+r_{t-i+1}|\tilde{r}_{t-1},...,\tilde{r}_{t-i+1}}(\tilde{r})dF_{r_{t+...+r_{t-i+1}|\tilde{r}_{t-1},...,\tilde{r}_{t-i+1}}(r). \]

If \( \tilde{r}_t \) is high then \( r_t \) is more likely to be high and the agent is likely to experience positive news utility. If the agent decides to ignore his portfolio, he will experience prospective news utility over the signal. In that case, prospective news utility in period \( t \) is

\[ \sum_{j=1}^{j_1} \beta^j p_{t+j} \alpha_{t-i}(1 + \gamma \sum_{\tau=1}^{T-t-j} \beta^\tau) \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \eta(\lambda - 1)(r - \tilde{r})dF_{r_{t+j+...+r_{t-i+1}}(r, \tilde{r}). \]

Here, \( p_{t+j} \) denotes the probability of looking up the portfolio in period \( t + j \) conditionally on period \( t \) information, such that \( \sum_{j=1}^{j_1} p_j = 1 \) that is, the investor knows he cannot delay looking up his portfolio beyond period \( t + j_1 \). For a simplified comparison, suppose the agent plans to look up his portfolio in period \( t + 1 \), so that \( j_1 = 1 \) and \( p_{j_1} = 1 \). In expectation, looking up the true return is more painful than merely experiencing prospective news utility simply because more uncertainty will be resolved. But, the difference between the two is smaller when the agent has received a more favorable signal. The reason is that expected marginal news utility from resolution of \( \epsilon_t \) is less if \( \tilde{r}_t \) is high because the agent considers news fluctuations to be high up on the utility curve. Accordingly, after having received a favorable signal, the agent is more likely to look up his portfolio. This behavior is commonly known as the “Ostrich effect” and is supported by empirical evidence (Karlsson et al. (2009)).

Nevertheless, this example also suggests that it is not unlikely that the agent will choose to ignore the signal and not adjust his attentiveness and consumption. After all, in the event of a favorable signal, he experiences news utility over the signal or news utility over the actual realization. Thus, the sole reason that he is more likely to look up the realization is the fact that the expected costs of receiving more information are lower conditional on a favorable signal. A simple simulation confirms this conclusion quantitatively. Figure 5 compares the news utility experienced over the signal to the expected news utility conditional on the signal. It can be seen that the quantitative difference is decreasing in the realization of the signal, but is very small in comparison to the overall variation in news utility.\(^{29}\) More details can be found in Appendix 5.4.

\(^{29}\)I choose a quantitative example that should produce a larger difference in experienced and expected news utility than the model under consideration. I choose, first, an annual horizon as a period’s length such that the signal has large information content, and, second, a variance of the signal that is equally variable as the stock-market variance because more noisy signals increase this difference in news utility. The parameter values are \( \mu = 8\% \), \( r_f = 2\% \), and \( \sigma_r = \sigma_\epsilon = 20\% \) and the preference parameters are \( \eta = 1 \) and \( \lambda = 2.5 \).
Figure 5: Comparison of experienced news utility over the signal and expected news utility conditional on the signal.

However, this comparison does not take into account the fact that the agent cares more about contemporaneous than prospective news that is, the expected news utility is weighted by $1 + \gamma \sum_{\tau=1}^{T-t} \beta^\tau$ while the prospective news utility is weighted by $\sum_{j=1}^{j} \beta^j p_{t+j}(1 + \gamma \sum_{\tau=1}^{T-t-j} \beta^\tau)$. But, if the agent’s horizon is long and the period is short the difference between the two is small. Moreover, if a period is short the contemporaneous realization of the return has only small quantitative implications for immediate consumption. Additionally, if a period’s length is short the agent compares experiencing news utility over a one-period realization of the signal to experiencing news utility over the multi-period uncertainty left from the previous inattentive periods. I confirm these conjectures by checking in the model simulation that the change in expected utilities in response to the presence of a signal does not affect the agent’s plan to look up his portfolio most of the time using a period’s length of one month and a relatively accurate signal with a standard deviation of $\sigma_x = \frac{\sigma_r^2}{2}$.

Overconfidence and extrapolative expectations. Now I ask how the agent’s behavior will be perceived by an outsider. The outsider acquires all available information because he does not experience any news utility over the agent’s portfolio or consumption. Will the outsider then perceive the agent’s behavior as overconfident or extrapolative? After all, whenever the agent decides to ignore his portfolio, his behavior, as reflected in his portfolio share, is based on a different information set than the outsider’s one. The agent’s information set in each period $t$ is denoted by $I_t$ and might contain today’s return $r_t$ and all past returns or only the returns past $r_{t-i}$. Even though the agent receives the signals $r_{t-i+1},..., r_t$, his behavior, as reflected in his portfolio share, will be based on the returns past $r_{t-i}$ only. This portfolio share is denoted by $f^\alpha(I_t)$. In contrast, the outsider’s information is denoted by $I_t^o$ and contains $r_t$ and all past $r_{t-i}$. 

33
I say that if $f^{a}(I_{t}) > f^{a}(I_{o})$ the agent will be perceived to be overconfident by an outsider. Whenever the agent ignores his portfolio on the grounds that the return realization is likely to be bad, he will have chosen a higher portfolio share just as if he expected high returns. Thus, an outsider would perceive his behavior as overconfident.

I say further that if $f^{a}(I_{t}) = \rho f^{a}(I_{o}) + (1 - \rho) f^{a}(I_{o - i})$ the agent will be perceived to have extrapolative expectations by an outsider. Whenever the agent decides to ignore his portfolio, but the outsider acquires all information, then the agent’s behavior, as reflected in his portfolio share, will be based on an outdated information set; thus, he appears to be extrapolating past returns. Overconfidence and extrapolative expectations are two descriptive theories about beliefs that have been assumed in a variety of behavioral-finance papers to explain stock prices: for example, Scheinkman and Xiong (2003), Malmendier and Tate (2005, 2008), Choi (2006), Hirshleifer and Yu (2011), and Barberis et al. (fthc).

6 Quantitative Implications for Life-Cycle Consumption and Portfolio Choice and Empirical Evidence

In Section 6.1, I assess the quantitative performance of the model with a structural estimation exercise using household portfolio data on participation and shares. I first choose an empirically plausible parametrization of the environmental parameters and the period’s length, in order to then estimate the preference parameters. For the parametrization, I explore how often the investor chooses to look up his portfolio given a plausible calibration of the environmental and preference parameters. I then use the implied average length of inattention as the period’s length in a standard life-cycle model. This standard model assumes power utility $u(c) = \frac{c^{1-\theta}}{1-\theta}$ with a coefficient of risk aversion $\theta$ instead of relying on log utility. The derivation is outlined in Appendix C.4. Section 6.1.1 quickly describes the household portfolio data, Section 6.1.2 presents results for different calibrations of the period’s length, Section 6.1.3 provides details on the identification, and Section 6.1.4 describes the estimation procedure and compares my estimates with those in the existing literature. Finally, in Section 6.2, I provide some suggestive empirical evidence for extensive rebalancing in portfolio choice using PSID household portfolio data.

6.1 Structural estimation

To validate the model quantitatively, I structurally estimate the preference parameters by matching the average empirical life-cycle profile of participation and portfolio shares using household portfolio data of the Survey of Consumer Finances (SCF) from 1992 to 2007.
6.1.1 Data

The SCF data is a statistical survey of incomes, balance sheets, pensions and other demographic characteristics of families in the United States, sponsored by the Federal Reserve Board in cooperation with the Treasury Department. The SCF was conducted in six survey waves from 1992 to 2007 but did not survey households consecutively. I construct a pseudo-panel by averaging participation and shares of all households at each age following the risk-free and risky-asset definitions of Flavin and Nakagawa (2008).

In addition to the age effects of interest, the data is contaminated by potential time and cohort effects, which constitutes an identification problem as time minus age equals cohort. In the portfolio-choice literature, it is standard to solve the identification problem by acknowledging age and time effects, as tradable and non-tradable wealth varies with age and contemporaneous stock-market happenings are likely to affect participation and shares, while omitting cohort effects (Campbell and Viceira (2002)). By contrast, in the consumption literature it is standard to omit time effects but acknowledge cohort effects (Gourinchas and Parker (2002)). I employ a new method, recently invented by Schulhofer-Wohl (2013), that solves the age-time-and-cohort identification problem with minimal assumptions. In particular, the method merely assumes that age, time, and cohort effects are linearly related. I first estimate a pooled OLS model, whereby I jointly control for age, time, and cohort effects and identify the model with a random assumption about its trend. Then, I estimate this arbitrary trend together with the structural parameters, which results in consistent estimates using data that is uncontaminated by time and cohort effects. This application of Schulhofer-Wohl (2013) to household portfolio data is an important contribution, as portfolio profiles are highly dependent on which assumptions the identification is based on, as is made clear by Ameriks and Zeldes (2004).

Figure 6 displays the uncontaminated empirical profiles for participation and portfolio shares as well as labor income. Both participation and portfolio shares are hump shaped over the life cycle. The predicted income profile is lower than the profile containing the disturbances because the SCF oversamples rich households but provides weights to adjust the regressions.
Figure 6: Participation, portfolio shares, and labor income over the life cycle.

The model’s quantitative predictions about consumption and wealth accumulation are also compared to the empirical profiles inferred from the Consumer Expenditure Survey (CEX). The CEX is a survey of the consumption expenditures, incomes, balance sheets, and other demographic characteristics of families in the United States, sponsored by the Bureau of Labor Statistics.

6.1.2 Calibrating the period’s length

I estimate a standard life-cycle model in which the investor looks up his portfolio each period. As can be inferred from the theoretical analysis, an important calibrational degree of freedom constitutes the model’s period length, which I determine now. As a first step, I will calibrate the risky and risk-free return moments, \( h\mu_0 \), \( \sqrt{h}\sigma_0 \), and \( hr^f_0 \), to a monthly investment horizon if \( h = 1 \). Then, \( h = 12 \) would recalibrate the model to an annual horizon. The literature suggests fairly tight ranges for the parameters of the log-normal return, \( 12\mu_0 = 8\% \), \( \sqrt{12}\sigma_0 = 20\% \), and the log risk-free rate, \( 12r^f_0 = 2\% \). Additionally, I choose for the agent’s horizon \( T = 60 \) years (720 months) and for his retirement period \( R = 10 \) years (120 months), in accordance with the life-cycle literature. Moreover, I set \( \eta = 1 \), \( \lambda = 2.5 \), and \( \gamma = 0.8 \), which are reasonable preference-parameter choices given the literature on prospect theory and reference dependence, as I will discuss in Section 6.1.4.

Under this calibration, I find that the agent looks up his portfolio approximately once a year early in life and chooses a zero portfolio share after the start of retirement. Figure 3 in Section 5.1.2 displays the news-utility and standard agents’ optimal portfolio shares. As can be seen, the news-utility agent’s share is increasing in the agent’s horizon and decreasing in the return realization. In contrast, the standard agent’s share is constant in the horizon and return realizations. Beyond these implications for portfolio choice, I look at the agent’s consumption profile as an additional check assuming that the period length is one year.
in line with the predictions of the analytical model and using the above parameter values and $\beta = 0.97$. As can be seen in Figure 7, the news-utility agent’s consumption profile is hump shaped whereas the standard agent’s consumption profile is increasing throughout. Not surprisingly, the standard agent accumulates wealth very rapidly, as his portfolio share is one. The news-utility consumption profile roughly matches the empirical consumption profile estimated from CEX data and also displayed in Figure 7.\(^{30}\)

![Figure 7: News-utility and standard agents’ consumption over the life cycle and empirical consumption profile.](image)

I conclude that a yearly investment horizon seems a reasonable calibrational choice; this has also been assumed by others in similar contexts (Benartzi and Thaler (1995) and Barberis et al. (2001)).

### 6.1.3 Identification

Are the empirical life-cycle participation and portfolio shares profiles able to identify the preference parameters? I am interested in five preference parameters, namely $\beta$, $\theta$, $\eta$, $\lambda$, and $\gamma$. As shown in Appendix C.4, both participation and portfolio shares are determined by the first-order condition $\gamma \frac{\partial \Phi_t}{\partial \alpha_t} (\eta F_{t-1}^{l}(A_t) + \eta \lambda (1 - F_{t}^{l-1}(A_t))) + \frac{\partial \psi_t}{\partial \alpha_t} = 0$ for which I observe the average of all households. $\frac{\partial \Phi_t}{\partial \alpha_t}$ represents future marginal consumption utility, as in the standard model, and is determined by $\beta$ and $\theta$, which can be separately identified in a finite-horizon model. $\frac{\partial \psi_t}{\partial \alpha_t}$ represents future marginal consumption and news utility and is thus determined by something akin to $\eta (\lambda - 1)$. $\eta F_{t}^{l-1}(A_t) + \eta \lambda (1 - F_{t}^{l-1}(A_t))$ represents the weighted sum of the cumulative distribution function of savings, $A_t$, of which merely the

\(^{30}\)Note that this empirical profile implicitly assumes that households do not retire, which is why consumption is not decreasing too much toward the end of life. I consider this comparison to be more adequate, as the model I use in this section abstracts from labor income.
average determined by $\eta 0.5(1 + \lambda)$ is observed. Thus, I have two equations in two unknowns and can separately identify $\eta$ and $\lambda$. Finally, $\gamma$ enters the first-order condition distinctly from all other parameters, and I conclude that the model is identified, which can also be verified by deriving the Jacobian that has full rank.

6.1.4 Estimation

Methods of simulated moments procedure. At an annual horizon, the literature suggests fairly tight ranges for the parameters of the log-normal return; I match these by estimating $\hat{\mu} - \hat{r}^f = 6.33\%$, $\hat{\sigma} = 19.4\%$, and the log risk-free rate, $\hat{r}^f = 0.86\%$, using value-weighted CRSP return data. Moreover, the life-cycle consumption literature suggests fairly tight ranges for the parameters determining stochastic labor income that is, labor income is log-normal, characterized by shocks with variance $\sigma_Y$, a probability of unemployment $p$, and a trend $G$ that I roughly match by estimating $\hat{\sigma}_Y = 0.2$, $p = 1\%$, and $\hat{G}_t$ from the SCF data. Labor income is correlated with the risky asset with a coefficient of approximately 0.2 following Viceira (2001) among others. Moreover, because 25 is chosen as the beginning of life by Gourinchas and Parker (2002), I choose $\hat{R} = 11$ and $\hat{T} = 54$ in accordance with the average retirement age in the US according to the OECD and the average life expectancy in the US according to the UN. After having calibrated the structural parameters governing the environment $\Xi = (\mu, \sigma, \sigma_Y, G, r^f, R, T)$, I now estimate the preference parameters $\theta = (\eta, \lambda, \gamma, \beta, \theta)$ by matching the simulated and empirical life-cycle profiles for portfolio shares. I focus on matching portfolio shares rather than participation, because the optimization procedure would prioritize them anyhow as they have lower variances than participation. The empirical profile is the average participation rate at each age $a \in [1, T]$ across all household observations $i$. More precisely, it is $\bar{\alpha}_a = \frac{1}{n_a} \sum_{i=1}^{n_a} \bar{\alpha}_{i,a}$ with $\bar{\alpha}_{i,a}$ being the household $i$’s portfolio share at age $a$ of which $n_a$ are observed. The theoretical population analogue to $\bar{\alpha}_a$ is denoted by $\alpha_a(\theta, \Xi)$ and the simulated approximation is denoted by $\hat{\alpha}_a(\theta, \Xi)$. Moreover, I define

$$\hat{g}(\theta, \Xi) = \hat{\alpha}_a(\theta, \Xi) - \bar{\alpha}_a.$$

In turn, if $\theta_0$ and $\Xi_0$ are the true parameter vectors, the procedure’s moment conditions imply that $E[g(\theta_0, \Xi_0)] = 0$. Let $W$ denote a positive definite weighting matrix. In that case,

$$q(\theta, \Xi) = \hat{g}(\theta, \Xi)W^{-1}\hat{g}(\theta, \Xi)^\prime$$

is the weighted sum of squared deviations of the simulated moments from their corresponding empirical moments. I assume that $W$ is a robust weighting matrix rather than the optimal weighting matrix to avoid small-sample bias. More precisely, I assume that $W$ corresponds
to the inverse of the variance-covariance matrix of each point of \( \bar{\alpha}_a \), which I denote by \( \Omega_g^{-1} \) and consistently estimate from the sample data. Taking \( \hat{\Xi} \) as given, I minimize \( q(\theta, \hat{\Xi}) \) with respect to \( \theta \) to obtain \( \hat{\theta} \) the consistent estimator of \( \theta \) that is asymptotically normally distributed with standard errors:

\[
\Omega_\theta = \left( G_\theta' W G_\theta \right)^{-1} G_\theta' W \left[ \Omega_g + \Omega_g \xi G_\xi \Omega_g' G_\theta W G_\theta' \left( G_\theta' W G_\theta \right)^{-1} \right].
\]

Here, \( G_\theta \) and \( G_\xi \) denote the derivatives of the moment functions \( \frac{\partial g(\theta_0, \xi_0)}{\partial \theta} \) and \( \frac{\partial g(\theta_0, \xi_0)}{\partial \xi} \), \( \Omega_g \) denotes the variance-covariance matrix of the second-stage moments, as above, that corresponds to \( E[g(\theta_0, \xi_0)g(\theta_0, \xi_0)'] \), and \( \Omega_g^s = \frac{n_a}{n_s} \Omega_g \) denotes the sample correction with \( n_s \) being the number of simulated observations at each age \( a \). As \( \Omega_g \), I can estimate \( \Omega_\xi \) directly and consistently from sample data. For the minimization, I employ a Nelder-Mead algorithm. For the standard errors, I numerically estimate the gradient of the moment function at its optimum. If I omit the first-stage correction and simulation correction the expression becomes \( \Omega_\theta = \left( G_\theta' \Omega_g^{-1} G_\theta \right)^{-1} \). Thus, I can test for overidentification by comparing \( \hat{g}(\hat{\theta}, \hat{\Xi}) \) to a chi-squared distribution with \( T - 5 \) degrees of freedom. The calibrated and estimated parameters can be found in Table 1.

### Calibration and Estimation Results

<table>
<thead>
<tr>
<th></th>
<th>( \hat{\mu} - \hat{\mu}^f )</th>
<th>( \hat{\sigma} )</th>
<th>( \hat{\sigma}_Y )</th>
<th>( \text{corr}(r, Y) )</th>
<th>( \hat{G}_t )</th>
<th>( p )</th>
<th>( \hat{r}^f )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>6.33%</td>
<td>19.4%</td>
<td>0.2</td>
<td>0.2</td>
<td></td>
<td>1%</td>
<td>0.86%</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>( \hat{\beta} )</th>
<th>( \hat{\theta} )</th>
<th>( \hat{\eta} )</th>
<th>( \hat{\lambda} )</th>
<th>( \hat{\gamma} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.987</td>
<td>1.81</td>
<td>0.87</td>
<td>3.03</td>
<td>0.87</td>
</tr>
<tr>
<td>(0.0087)</td>
<td>(0.0084)</td>
<td>(0.0065)</td>
<td>(0.0059)</td>
<td>(0.0079)</td>
</tr>
</tbody>
</table>

Table 1: This table displays the calibrated and estimated parameters as well as the standard errors of the estimated parameters (in parentheses). The overidentification test result is 16.07 and the critical value is 69.83.

**Discussion of estimation results.** I refer to the literature regarding the standard estimates for \( \beta \) and \( \theta \), but I discuss in more detail the plausibility of the news-utility parameter values, \( \eta \), \( \lambda \), and \( \gamma \). In the following, I demonstrate that my estimates are consistent with existing micro evidence on risk and time preferences. In Table 2 in Appendix A, I illustrate the risk preferences of the news-utility and standard agents regarding gambles with various stakes. In particular, I calculate the required gain \( G \) for a range of losses \( L \) to make each agent indifferent between accepting or rejecting a 50/50 either win \( G \) or lose \( L \) gamble at a wealth level of 300,000 in the spirit of Rabin (2001) and Chetty and Szeidl (2007).\(^{31}\)

\(^{31}\)In a canonical asset-pricing model, Pagel (2012) demonstrates that news-utility preferences constitute an additional step towards resolving the equity-premium puzzle, as they match the historical level and the
First, I want to match risk attitudes towards gambles regarding immediate consumption, which are determined solely by $\eta$ and $\lambda$ because it can be reasonably assumed that utility over immediate consumption is linear. Thus, $\eta \approx 1$ and $\lambda \in [2, 3]$ are suggested by the laboratory evidence on loss aversion over immediate consumption, most importantly, the endowment effect literature. $\eta(\lambda - 1) \approx 2$ implies that the equivalent Kahneman and Tversky (1979) coefficient of loss aversion is around 2, because the news-utility agent experiences consumption and news utility, whereas classical prospect theory effectively consists of news utility only, and consumption utility works in favor of any small-scale gamble.\textsuperscript{32} In Table 2, it can be seen that news-utility preferences generate reasonable attitudes towards small and large gambles over consumption and wealth. In contrast, the standard agent is risk neutral for small and medium stakes and reasonably risk averse for large stakes only. I elicit the agents’ risk attitudes towards wealth gambles by assuming that each of them is presented with the gamble after all consumption in that period has taken place. The news-utility agent will only experience prospective news utility over the wealth gambles’ outcomes that is determined by the prospective news discount factor $\gamma$. $\gamma$ implies attitudes towards intertemporal consumption trade-offs that are similar to those implied by a hyperbolic-discounting parameter of the same value. Empirical estimates for the hyperbolic-discounting parameter in a variety of contexts typically range between 0.7 and 0.8 (e.g., Laibson et al. (2012)). Thus the experimental and field evidence on peoples’ attitudes towards intertemporal consumption trade-offs suggests that $\gamma \approx 0.8$ and $\beta \approx 1$ are plausible estimates. Moreover, my estimates match the parameter values obtained by a structural estimation of a life-cycle consumption model in Pagel (2013).

Beyond matching micro estimates, the implied life-cycle profiles of shares and participation roughly match the hump-shaped empirical profiles. Portfolio shares and participation are both lower than in the standard model because news-utility preferences retain first-order risk aversion even in the presence of background risk, as shown in Section 3.1.2, so long as labor income is not too dispersed. While portfolio shares are matched, participation ends up being higher in the model than in the data although the general profiles correspond. The profiles are decreasing in the end of life because risk is not as well diversified across time and because the agent’s expected labor income is decumulating. The profiles are increasing in variation of the equity premium while simultaneously implying plausible attitudes towards small and large wealth bets.

\textsuperscript{32}For illustration, I borrow a concrete example from Kahneman et al. (1990), in which the authors distribute a good (mugs or pens) to half of their subjects and ask those who received the good about their willingness to accept (WTA) and those who did not receive it about their willingness to pay (WTP) if they traded the good. The median WTA is $5.25, whereas the median WTP is $2.75. Accordingly, I infer $(1 + \eta)u(mug) = (1 + \eta \lambda)2.25$ and $(1 + \eta \lambda)u(mug) = (1 + \eta)5.25$, which implies that $\lambda \approx 3$ when $\eta \approx 1$. I obtain a similar result for the pen experiment.
the beginning of life because the labor income profile is hump shaped and I assume a low but positive correlation between labor-income and stock-market risk. Increasing expected labor income makes labor-income and stock-market risks more bearable, because news utility is proportional to consumption utility, so that fluctuations in good and bad news hurt less on the flatter part of the concave utility curve. That portfolio shares can be increasing at the beginning of life when income growth is high for high levels of risk aversion has first been noted by Viceira (2001) and Cocco et al. (2005). Here, it also helps that the news-utility agent’s savings profile is relatively flat but not too low at the beginning of life, as he has a first-order precautionary-savings motive, because savings tend to negatively correlate with the portfolio share as in the standard model. Moreover, the implied life-cycle consumption profile displays a hump similar to what I find in the CEX data.

6.2 Testing the model’s implications using household portfolio data

In the following, I provide some suggestive empirical evidence for extensive rebalancing in portfolio choice. I use the same data set as Brunnermeier and Nagel (2008) from the PSID, which contains household characteristics, wealth, income, stock market holdings, business equity, and home equity. Brunnermeier and Nagel (2008) aim to test if portfolio shares are increasing in wealth as would be predicted by a habit-formation model and other asset-pricing models that rely on increasing risk aversion to explain countercyclical equity premia. The authors find that, if anything, the risky asset share is slightly decreasing in wealth. In the following, I show that the risky asset share seems to be decreasing in the innovation to wealth for the group of households that adjust their risky asset holdings, as predicted by the news-utility model. Nevertheless, in an asset-pricing model, news utility predicts countercyclical equity premia as shown in Pagel (2012) too.

I follow the methodology of Brunnermeier and Nagel (2008). However, instead of the risky asset share’s response to changes in wealth, I am interested in its response to innovations or unexpected changes in wealth. The unexpected change in wealth, \( \Delta w_t \), corresponds to the residual of a predictive regression of the change in wealth on all other variables used by Brunnermeier and Nagel (2008) including last period’s change and level in wealth, i.e.,

\[
\Delta w_t = \beta q_{t-2} + \gamma \Delta h_t + \tilde{\Delta} w_t.
\]

\( q_{t-2} \) consists of a vector of ones and constant or lagged variables that are known at date \( t \), such as age, education, gender, marital status, employment, inheritances, etc.\(^{33}\) \( \Delta h_t \) is a

\(^{33}\)Following Brunnermeier and Nagel (2008), I include age and age\(^2\); indicators for completed high school and college education, respectively, and their interaction with age and age\(^2\); dummy variables for gender and
vector of variables that captures major changes in family composition or asset ownership, such as changes in family size, house ownership, etc. Moreover, I restrict the regression to those households who did change their risky asset holdings, as the model predicts that some people are inattentive. Furthermore, Brunnermeier and Nagel (2008) analyze the change in the risky asset share, whereas I use the level of the risky asset share as the dependent variable and the lagged risky asset share as an additional independent variable. I restrict myself to the second subsample that contains the PSID waves of 1999, 2001, and 2003, as the first subsample has a five year difference, which is likely to be too long for analyzing an expectations-based reference point. I run a pooled regression of the form

$$\alpha_t = b\alpha_{t-2} + \beta q_{t-2} + \gamma \Delta h_t + \rho \Delta w_t + \epsilon_t$$

with $\alpha_{t-2}$ denoting the lagged risky asset share. Consistent with the results of Brunnermeier and Nagel (2008), the coefficient on the unexpected change in wealth for those households who did change their risky asset holdings in that period is negative but relatively small. The coefficient is more negative and significant if I restrict the sample to households that changed their risky asset holdings, consistent with the model. I obtain a coefficient of approximately -.13 with a t-statistic of 3.34 which implies that an unexpected decrease in wealth of 20% leads to an increase in the risky asset share by 2.6%. However, the model predicts a much larger effect of an innovation in wealth by $\sigma r$, i.e., $e^{\mu_r + \sigma r} - e^{\mu_r} \approx 20\%$, yields to a reduction in the risky asset share by roundabout 10% when I omit the variation in $1 - \rho_t$. Alternatively, I run a 2SLS regression in which I instrument the unexpected change in wealth by the unexpected change in labor income using the same methodology as Brunnermeier and Nagel (2008) obtaining similar results.

Following Brunnermeier and Nagel (2008), I include changes in some household characteristics between $t - 2$ and $t$: changes in family size, changes in the number of children, and a sets of dummies for house ownership, business ownership, and non-zero labor income at $t$ and $t - 2$.  

34Following Brunnermeier and Nagel (2008), I include changes in some household characteristics between $t - 2$ and $t$: changes in family size, changes in the number of children, and a sets of dummies for house ownership, business ownership, and non-zero labor income at $t$ and $t - 2$.  

35Following Brunnermeier and Nagel (2008), I define liquid assets as the sum of holdings of stocks and mutual funds plus riskless assets, where riskless assets are defined as the sum of cash-like assets and holdings of bonds. Subtracting other debts, which comprises non-mortgage debt such as credit card debt and consumer loans, from liquid assets yields liquid wealth. I denote the sum of liquid wealth, equity in a private business, and home equity as financial wealth. I then calculate the risky asset share as the sum of stocks and mutual funds, home equity, and equity in a private business, divided by financial wealth. Alternatively, I could exclude home equity and equity in a private business.
7 Discussion and Conclusion

This paper provides a comprehensive analysis of the portfolio implications of news utility, a recent theoretical advance in behavioral economics which successfully explains micro evidence in a broad range of domains. The preferences’ robust explanatory power in other domains is important because I put emphasis on potentially normative questions. In particular, I ask how often should people look up and rebalance their portfolios. Several of my results are applicable to financial advice. First, the preferences predict that people prefer to stay inattentive and can diversify over time. Therefore, people should look up their portfolios only once in a while, should choose lower portfolio shares toward the end of life, and should delegate the management of their portfolios to an agent who encourages inattention and rebalances actively. Furthermore, the preferences make specific predictions about how investors should rebalance their portfolios if they look them up. Most importantly, people should choose a lower portfolio share after the market goes up and thus follow a buy-low-sell-high investment strategy.

The intuitions behind my results are immediately appealing. If the investor cares more about bad news than good news, then fluctuations in expectations about future consumption hurt on average. Thus, the investor prefers to be inattentive most of the time and does not rebalance his portfolio. Once in a while, however, the investor has to look up his portfolio and then rebalances extensively. After the market goes down, the investor finds it optimal to increase his portfolio share temporarily so as not to realize all the bad news associated with future consumption. Hereby, the investor effectively delays the realization of bad news until the next period, by which point his expectations will have decreased. On the other hand, after the market goes up, the investor finds it optimal to play safe and wants to realize the good news by liquidating his asset holdings. The investor may ignore his portfolio because he has access to two different accounts: a brokerage account, through which he can invest a share of his wealth into the stock market, and a checking account, which finances his inattentive consumption. These two accounts relate to the notion of mental accounting because the accounts finance different types of consumption, they feature different marginal propensities to consume, the investor treats windfall gains in each differently, and they allow the investor to overconsume less and to exercise more self control.

Theoretically, I obtain analytical results to explain these phenomena under the assumption of log utility and the Campbell and Viceira (2002) approximation of log portfolio returns. Quantitatively, I structurally estimate the preference parameters and show that the model’s predictions match the empirical life-cycle evidence on participation, portfolio shares, consumption, and wealth accumulation and provide some suggestive evidence for extensive
rebalancing. In the future, I would like to further explore cross-sectional asset pricing, as the theory predicts that more newsy investments should carry higher risk premia.

References


A  Attitudes over Wealth Bets

<table>
<thead>
<tr>
<th>Loss (L)</th>
<th>Standard Contemp.</th>
<th>News-Utility Prospective</th>
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Table 2: For each loss L, the table’s entries show the required gain G to make each agent indifferent between accepting and rejecting a 50-50 gamble win G or lose L at a wealth level of 300,000.

B  Derivations of the portfolio share approximation in the presence of labor income

I first derive the news-utility agent’s optimal portfolio share in greater detail. To obtain a closed-form portfolio solution, I assume log utility $u(c) = \log(c)$. In general, logs are denoted by lower case letters. The news-utility agent wants to maximize

$$E[\log(W) + (\log(R^f + \alpha(R-R^f))) + \eta(\lambda - 1) \int_R \log(R^f + \alpha(R-R^f)) - \log(R^f + \alpha(\tilde{R}-R^f))) dF_R(\tilde{R})]$$

or equivalently

$$E[(r^f + \alpha(r-r^f) + \alpha(1-\alpha)\frac{\sigma^2}{2}) + \eta(\lambda - 1) \int_r \log(R^f + \alpha(R-R^f)) - \log(R^f + \alpha(\tilde{R}-R^f))) dF_R(\tilde{R})]$$

$$\Rightarrow E[(r^f + \alpha(r-r^f) + \alpha(1-\alpha)\frac{\sigma^2}{2}) + \alpha\eta(\lambda - 1) \int_r (r - \tilde{r})dF_r(\tilde{r})]$$

$$\Rightarrow E[(r^f + \alpha(r-r^f) + \alpha(1-\alpha)\frac{\sigma^2}{2}) + \alpha E[\eta(\lambda - 1) \int_r (r - \tilde{r})dF_r(\tilde{r})]]$$

which results in the following first-order condition

$$\mu - r^f - \alpha\sigma^2 + E[\eta(\lambda - 1) \int_r (r - \tilde{r})dF_r(\tilde{r})] = 0$$

$$\alpha^* = \frac{\mu - r^f + E[\eta(\lambda - 1) \int_r (r - \tilde{r})dF_r(\tilde{r})]}{\sigma^2}. $$
I now consider the case in which labor income is riskless, i.e., $Y > 0$ and $F_Y$ degenerate. In this case, I can simply transform the problem to the one above by noting that $Y$ dollars of labor income is equivalent to $\frac{1}{\rho^2}Y$ dollars invested in the risk-free asset. Thus, the agent wants to invest the fraction $\alpha^*$ out of his transformed wealth $W + \frac{1}{\rho^2}Y$ in the risky asset; accordingly, his actual share invested into the risky asset is

$$\alpha^*_{Y>0} = \frac{\alpha^*(W + \frac{1}{\rho^2}Y)}{W} = \alpha^*(1 + \frac{Y}{\rho^2W}).$$

Now, I move on to stochastic labor income. Stochastic income makes the model considerably more complicated, as its solution requires numerical techniques. In order to provide analytical insights into the model’s mechanisms, I will continue to follow Campbell and Viceira (2002) and employ an approximation strategy for the log portfolio return, the consumption function, and the agent’s first-order condition. More specifically, the log portfolio return $r^p$ is approximated by $r^f + \alpha(r - r^f) + \alpha(1 - \alpha)\frac{\sigma^2}{2}$ and the log of the consumption function $C = g^C(r, y) = W(e^{r^f} + \alpha(e^r - e^{r^f})) + e^y$ can be approximated by

$$\log(C) - \log(Y) = c - y = \log(e^{w+r^p-y} + 1) \Rightarrow c \approx k + \rho(w + r^p) + (1 - \rho)y$$

with $\rho$ and $k$ being log-linearization parameters as in Campbell and Viceira (2002). The agent’s first-order condition is given by

$$E[e^{-c}(e^r - e^{r^f}) + \eta(\lambda - 1) \int_y^\infty \int_r^\infty (e^{-c}(e^r - e^{r^f}) - e^{-\log(g^C(\tilde{r}, \tilde{y}))(e^\tilde{r} - e^{r^f})})dF_r(\tilde{r})dF_y(\tilde{y})] = 0.$$

The first part of this equation can be rewritten as the following approximation

$$E[e^{-c}(e^r - e^{r^f})] = 0 \Rightarrow E[e^{-(\rho\sigma + (1-\rho)y) + r}] - E[e^{-(\rho\sigma + (1-\rho)y) + r^f}] = 0$$

$$\Rightarrow E[e^{(1-\rho\sigma)r-(1-\rho)y}] - E[e^{-\rho\sigma - (1-\rho)y + r^f}] = 0$$

$$e^{(1-\rho\sigma)(\mu - \frac{1}{2}\sigma^2) - (1-\rho)\mu_y + 0.5(1-\rho)\sigma^2 + 0.5(1-\rho)\sigma_y^2 - (1-\rho)\sigma^2 - (1-\rho)\sigma_y^2}$$

$$- e^{-\rho\sigma(\mu - \frac{1}{2}\sigma^2) - (1-\rho)\mu_y + 0.5(1-\rho)\sigma^2 + 0.5(1-\rho)\sigma_y^2 + \rho\sigma(1-\rho)\sigma^2 + r^f} = 0$$

$$e^{\mu - \rho\sigma^2 - (1-\rho)\sigma_y^2 - r^f} = 1 \Rightarrow \mu - r^f - \rho\sigma^2 - (1-\rho)\sigma_y^2 = 0$$

therefore, the news-utility first-order condition can be rewritten as the following approximation

$$e^{\mu - \rho\sigma^2 - (1-\rho)\sigma_y^2 - r^f} - 1 + e^{\rho\sigma(\mu - \frac{1}{2}\sigma^2) + (1-\rho)\mu_y - 0.5(1-\rho)\sigma^2 - 0.5(1-\rho)\sigma_y^2 - (1-\rho)\sigma^2 - (1-\rho)\sigma_y^2}$$

$$\eta(\lambda - 1)E[\int_r^\infty \int_y^\infty (e^{-\rho\sigma r - (1-\rho)y}(e^r - r^f) - 1) - e^{-\rho\sigma r - (1-\rho)y}(e^r - r^f) - 1) dF_r(\tilde{r})dF_y(\tilde{y})] = 0.$$
and if I approximate $e^x \approx 1 + x$ for $x$ small I end up with

$$
\mu - \rho \alpha \sigma^2 - (1 - \rho) \sigma_{ry} - r^f + \eta(\lambda - 1) E[\int_s^\infty (s - \bar{s}) dF(s)] = 0.
$$

This first-order condition results in the portfolio share in the text.

I now illustrate the agent’s first-order risk aversion in the presence of background risk without relying on the approximation above. To simplify the exposition, suppose the agent’s consumption is $\alpha r + y$ with $r \sim F_r$ and $y \sim F_y$ independently distributed, his risk premium for investing into the risky return $r$ is then given by

$$
\pi = \log(\alpha E[r] + y) + E[\eta(\lambda - 1) \int_y^\infty (\log(\alpha E[r] + y) - \log(\alpha E[r] + \bar{y})) dF_y(\bar{y})]
$$

$$
- E[\log(\alpha r + y) + \eta(\lambda - 1) \int_r^\infty \int_y^\infty (\log(\alpha r + y) - \log(\alpha \bar{r} + \bar{y})) dF_y(\bar{y}) dF_r(\bar{r})]
$$

and it’s marginal value for an additional increment of risk is

$$
\frac{\partial \pi}{\partial \alpha} = E[\frac{E[r]}{\alpha E[r] + y} + \eta(\lambda - 1) \int_y^\infty (\frac{E[r]}{\alpha E[r] + y} - \frac{E[r]}{\alpha E[r] + \bar{y}}) dF_y(\bar{y})]
$$

$$
- E[\frac{r}{\alpha r + y} + \eta(\lambda - 1) \int_r^\infty \int_y^\infty (\frac{r}{\alpha r + y} - \frac{\bar{r}}{\alpha \bar{r} + \bar{y}}) dF_y(\bar{y}) dF_r(\bar{r})].
$$

Now, what happens if return risk becomes small, i.e., $\alpha \to 0$

$$
\frac{\partial \pi}{\partial \alpha} \bigg|_{\alpha=0} = \eta(\lambda - 1) E[\int_y^\infty (\frac{E[r]}{y} - \frac{E[r]}{\bar{y}}) dF_y(\bar{y})] - E[\eta(\lambda - 1) \int_r^\infty \int_y^\infty (\frac{r}{y} - \frac{\bar{r}}{\bar{y}}) dF_y(\bar{y}) dF_x(\bar{x})].
$$

For the standard agent $\frac{\partial \pi}{\partial \alpha} \bigg|_{\alpha=0} = 0$, which implies that he is second-order risk averse. For the news-utility agent, the first integral is necessarily positive and dominates the second integral that can be negative or positive as in the second integral the positive effect of $r$ enters on top of the negative effect of $y$. 

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C Derivation of the Inattentive Life-Cycle Portfolio Choice Model

C.1 The monotone-personal equilibrium

I follow a guess and verify solution procedure. The agent adjusts his portfolio share and consumes a fraction $\rho_t$ out of his wealth if he looks up his portfolio and a fraction $\rho_t^i$ out of his wealth if he stays inattentive. Suppose he last looked up his portfolio in period $t - i$, i.e., he knows $W_{t-i}$. And suppose he will look up his portfolio in period $t + j_1$ but is inattentive in period $t$. Then his inattentive consumption in periods $t - i + 1$ to $t + j_1 - 1$ is given by (for $k = 1, ..., i + j_1 - 1$)

$$C^i_{t-i+k} = (W_{t-i} - C_{t-i})(R^d)^k \rho_{t-i+k}^i$$

and his consumption when he looks up his portfolio in period $t + j_1$ is given by

$$C_{t+j_1} = W_{t+j_1} \rho_{t+j_1} = (W_{t-i} - C_{t-i} - \sum_{k=1}^{i+j_1-1} C^i_{t-i+k})(R^d)^{i+j_1} + \alpha_{t-i}(\prod_{j=1}^{i+j_1} R_{t-i+j} - (R^f)^{i+j_1})\rho_{t+j_1}$$

Now, suppose the agent looks up his portfolio in period $t$ and then chooses $C_t$ and $\alpha_t$ knowing that he will look up his portfolio in period $t + j_1$ next time. I first explain the optimal choice of $C_t$. First, the agent considers marginal consumption and contemporaneous marginal news utility given by

$$u'(C_t)(1 + \eta F_r(r_t)F_r(r_{t-i+1}) + \eta \lambda (1 - F_r(r_t)F_r(r_{t-i+1})))$$

To understand these terms note that the agent takes his beliefs as given, his admissible consumption function $C_t$ is increasing in $r_t + ... + r_{t-i+1}$ such that $F^t_{C_t-i}(C_t) = F_r(r_t)F_r(r_{t-i+1})$, and

$$\frac{\partial(\eta \int_C u(C_t) - u(x))dF^t_{C_t-i}(x) + \eta \lambda \int_C u(C_t) - u(x))dF^t_{C_t}(x) }{\partial C_t} = u'(C_t)(\eta F^t_{C_t-i}(C_t) + \eta \lambda (1-F^t_{C_t-i}(C_t))).$$

Second, the agent takes into account that he will experience prospective news utility over all consumption in periods $t + 1, ..., T$. Inattentive consumption in periods $t + 1$ to $t + j_1 - 1$ is as above given by (for $k = 1, ..., j_1 - 1$)

$$C^i_{t+k} = (W_t - C_t)(R^d)^k \rho_{t+k}^i$$

and thus proportional to $W_t - C_t$. Attentive consumption in period $t + j_1$ is given by

$$C_{t+j_1} = W_{t+j_1} \rho_{t+j_1} = (W_t - C_t - \sum_{k=1}^{j_1-1} C^i_{t+k})(R^f)^{j_1} + \alpha_t(\prod_{j=1}^{j_1} R_{t+j} - (R^f)^{j_1})\rho_{t+j_1}$$
and thus proportional to $W_t - C_t$. As in the derivation in Section 5.2, prospective marginal news utility is

$$
\frac{\partial (\gamma \sum_{j=1}^{j_1-1} \beta^j n(F_{t,t-i}^t(W_t - C_t) R_t^i \sqrt{\rho_{k+i}^t})) + \gamma \sum_{j=1}^{j_1-1} \beta^j n(F_{t+1}^{t,i}(W_t - C_t))}{\partial C_t}
$$

$$
= \frac{\partial \log(W_t - C_t)}{\partial C_t} \sum_{j=1}^{t-t} \beta^j (\eta F_t(r_t) \ldots F_t(r_{t-i+1}) + \eta \lambda (1 - F_t(r_t) \ldots F_t(r_{t-i+1}))).
$$

To understand this derivation note that the agent takes his beliefs as given, future consumption is increasing in today’s return realization, and the only terms that realize and thus do not cancel out of the news-utility terms are $W_t - C_t$ such that $F_{t+1}^{t,i}(W_t - C_t) = F_t(r_t) \ldots F_t(r_{t-i+1})$. As an example consider the derivative of prospective news-utility in period $t + 1$

$$
\frac{\partial \beta n(F_{t+1}^{t-i}(W_t - C_t))}{\partial C_t} = \frac{\partial \beta}{\partial C_t} \frac{\mu((W_t - C_t) R_t^i \sqrt{\rho_{k+i}^t}) - \log(x)) dF_{t+1}^{t-i}(W_t - C_t)}{\partial C_t}
$$

$$
= -u'(W_t - C_t) \frac{\beta}{\partial C_t} \left( \sum_{j=1}^{t-t} \beta^j E_t[\log(C_{t+j})] \right) = -u'(W_t - C_t) \sum_{j=1}^{t-t} \beta^j
$$

Putting the three pieces together, optimal consumption if the agent looks up his portfolio in period $t$ is determined by the following first-order condition

$$
u'(C_t)(1 + \eta F_t(r_t) \ldots F_t(r_{t-i+1}) + \eta \lambda (1 - F_t(r_t) \ldots F_t(r_{t-i+1})))
$$

$$
-\gamma u'(W_t - C_t) \sum_{j=1}^{t-t} \beta^j (\eta F_t(r_t) \ldots F_t(r_{t-i+1}) + \eta \lambda (1 - F_t(r_t) \ldots F_t(r_{t-i+1}))) - u'(W_t - C_t) \sum_{j=1}^{t-t} \beta^j = 0.
$$

In turn, the solution guess can be verified

$$
C_t
$$

$$
W_t = \rho_t = \frac{1}{1 + \sum_{j=1}^{t-t} \beta^j (1 + \gamma (\eta F_t(r_t) \ldots F_t(r_{t-i+1}) + \eta \lambda (1 - F_t(r_t) \ldots F_t(r_{t-i+1})))}
$$
The optimal portfolio share depends on prospective news utility in period \(t\) for all the consumption levels in periods \(t + j_1\) to \(T\) that are all proportional to \(W_{t + j_1}\) (that is determined by \(\alpha_t\)). Moreover, the agent’s consumption and news utility in period \(t + j_1\) matters. However, consumption in inattentive periods \(t\) to \(t + j_1 - 1\) depends only on \(W_t - C_t\), which is not affected by \(\alpha_t\). Moreover, \(\alpha_t\) cancels in all expected news-utility terms after period \(t + j_1\). Thus, the relevant terms in the maximization problem for the portfolio share are given by

\[
\sum_{\tau = 0}^{T - t - j_1} \beta^{\tau} n(F_{C_{t+j_1+\tau}}^{l,t-i}) + \beta^{j_1} E_t[n(C_{t+j_1}, F_{C_{t+j_1}}^{t,j_1,t}) + \gamma \sum_{\tau = 1}^{T - t - j_1} \beta^{\tau} n(F_{C_{t+j_1+\tau}}^{t,j_1,t}) + j_1 \sum_{\tau = 0}^{T - t - j_1} \beta^{\tau} \log(C_{t+j_1+\tau})]
\]

The derivative of the first term is

\[
\frac{\partial \gamma^{j_1} \sum_{\tau = 0}^{T - t - j_1} \beta^{\tau} n(F_{C_{t+j_1+\tau}}^{l,t-i})}{\partial \alpha_t} = j_1 (\mu - r^f + \alpha_t \sigma^2) \gamma^{j_1} \sum_{\tau = 0}^{T - t - j_1} \beta^{\tau} (\eta F_r(r_t) \ldots F_r(r_{t-i+1}) + \eta \lambda (1 - F_r(r_t) \ldots F_r(r_{t-i+1}))).
\]

To illustrate where the derivative comes from

\[
\log(C_{t+j_1}) = \log\left(W_{t+j_1} \rho_{t+j_1}\right) = \log\left(W_t - C_t\right) \left(1 - \sum_{j=1}^{j_1-1} \rho_{t+j}^{in}\right) \left(\prod_{j=1}^{j_1} R_{t+j} - (R^f)^{j_1}\right) \left(1 - \rho_{t+j_1}\right) R^d_{t+j_1+1}
\]

thus the only term determined by \(\alpha_t\) is \(\log\left((R^f)^{j_1} + \alpha_t \left(\prod_{j=1}^{j_1} R_{t+j} - (R^f)^{j_1}\right)\right) \approx j_1 r^f + \alpha_t \left(\sum_{j=1}^{j_1} r_{t+j} - j_1 r^f\right) + j_1 (1 - \alpha) \sigma^2\) and the only terms that are different from the agent’s prior beliefs are \((W_t - C_t)\) (and the agent takes his beliefs as given and future consumption is increasing in today’s return as in the derivation of the consumption share above). Moreover,

\[
C_{t+j_1+1}^{in} = (W_t - C_t) \left(1 - \sum_{j=1}^{j_1-1} \rho_{t+j}^{in}\right) \left(\prod_{j=1}^{j_1} R_{t+j} - (R^f)^{j_1}\right) \left(1 - \rho_{t+j_1}\right) R^d_{t+j_1+1}
\]

thus the only term determined by \(\alpha_t\) are \(\log\left((R^f)^{j_1} + \alpha_t \left(\prod_{j=1}^{j_1} R_{t+j} - (R^f)^{j_1}\right)\right)\) and the only terms that are different from the agent’s prior beliefs are \((W_t - C_t)\). Thus, in the derivation the term \(j_1 (\mu - r^f + \alpha_t \sigma^2)\) is left as the agent does not consider his beliefs in the optimization and the integrals are determined by \(F_{W_t-C_t}^{l-i}(W_t - C_t) = F_r(r_t) \ldots F_r(r_{t-i+1}).\)

As explained above, the derivative of the continuation value is

\[
\frac{\beta^{j_1} E_t[n(C_{t+j_1}, F_{C_{t+j_1}}^{t,j_1,t}) + \gamma \sum_{\tau = 1}^{T - t - j_1} \beta^{\tau} n(F_{C_{t+j_1+\tau}}^{t,j_1,t}) + \beta^{j_1} \sum_{\tau = 0}^{T - t - j_1} \beta^{\tau} \log(C_{t+j_1+\tau})]}{\partial \alpha_t}
\]

\[
= \beta^{j_1} (1 + \gamma \sum_{\tau = 1}^{T - t - j_1} \beta^{\tau}) \sqrt{j_1} \sigma E[\eta (\lambda - 1) \int_{\bar{s}}^{\infty} (s - \bar{s})dF(\bar{s})] + \beta^{j_1} \sum_{\tau = 0}^{T - t - j_1} \beta^{\tau} j_1 (\mu - r^f - \alpha_t \sigma^2).
\]
Altogether, the optimal portfolio share is given by

$$
\alpha_t = \frac{\mu - r F_t + \frac{\beta^1 (1 + \gamma \sum_{\tau=t-j_1}^{t-1} \beta^\tau) \sqrt{1 + \frac{\beta^1 \sum_{\tau=t-j_1}^{t-1} \beta^\tau}{1 + \gamma (\eta F_t(r_t) \ldots F_t(r_{t-i-1}) + \eta \lambda (1 - F_t(r_t) \ldots F_t(r_{t-i-1})))}}}{\sigma^2}.
$$

The sufficient condition for an optimum is satisfied only if $\alpha_t \geq 0$; if $\alpha_t < 0$, the agent would choose $\alpha_t = 0$ instead. If the agent is inattentive in period $t$, his consumption is determined by the following first-order condition

$$
u'(C_{t-i}^{in}) - \sum_{\tau=0}^{T-t-j_1} \beta^{j_1 + \tau} u'(W_{t-i} - C_{t-i}) - \sum_{k=1}^{i+j_1-1} \frac{C_{t-i+k}}{(R^d)^k} \frac{1}{(R^d)^i} = 0.
$$

The term concerning consumption in period $t$ is self-explanatory. The terms concerning consumption from periods $t - i$ to $t + j_1 - 1$ drop out as they are determined by the solution guess $C_{t-i}^{in} = (W_{t-i} - C_{t-i})(R^d)^i \rho_{t-i}^{in}$. The terms concerning consumption from period $t + j_1$ on are all proportional to $log(W_{t+j_1})$ which equals $log(W_{t+i} - C_{t-i} - \sum_{k=1}^{i+j_1-1} C_{t-i+k}^{in} / (R^d)^k)$ plus the log returns from period $t - i + 1$ to period $t + j_1$, which, however, drop out by taking the derivative with respect to $C_{t-i}^{in}$. Accordingly,

$$
\frac{1}{C_{t-i}^{in}} - \sum_{\tau=0}^{T-t-j_1} \beta^{j_1 + \tau} \frac{1}{W_{t-i} - C_{t-i} - \sum_{k=1}^{i+j_1-1} \frac{C_{t-i+k}}{(R^d)^k}} \frac{1}{(R^d)^i} = 0 \Rightarrow \frac{1}{\rho_{t-i}^{in}} = \sum_{\tau=0}^{T-t-j_1} \beta^{j_1 + \tau} \frac{1}{1 - \sum_{k=1}^{i+j_1-1} \rho_{t-i+k}^{in}}
$$

And the solution guess $(C_{t-i}^{in} = (W_{t-i} - C_{t-i})(R^d)^i \rho_{t-i}^{in})$ can be verified. The agent does not deviate and overconsume in inattentive periods $t - i + 1$ to $t - 1$ so long as the bad news from deviating outweigh the good news from doing so, i.e., $u'(R^d)^k \rho_{t-i+k}^{in}) (1 + \eta) < (\beta R^d)^j u'((R^d)^{k+j} \rho_{t-i+k+j}^{in}) (1 + \eta \lambda) \Rightarrow u'(\rho_{t-i+k}^{in}) (1 + \eta) < \beta^j u'(\rho_{t-i+k+j}^{in}) (1 + \gamma \eta \lambda)$ for $k = 1, ..., i - 2$ and $j = 1, ..., i - k - 1$. In the derivation, I assume that this condition holds so that, in inattentive periods, the agent does not deviate from his consumption path and does not overconsume time inconsistently. For $\beta \approx 1$ and given that $\rho_{t-i+k}^{in} \approx \rho_{t-i+k+j}^{in}$, this condition is roughly equivalent to $\lambda > \frac{1}{\gamma}$.

C.2 The monotone-precommitted equilibrium

Suppose the agent has the ability to pick an optimal history-dependent consumption path for each possible future contingency in period zero when he does not experience any news utility. Thus, in period zero the agent chooses optimal consumption in period $t$ in each
possible contingency jointly with his beliefs, which of course coincide with the agent’s optimal state-contingent plan. For instance, consider the joint optimization over consumption and beliefs for $C(Y^*)$ when income $Y^*$ has been realized

$$
\frac{\partial}{\partial C(Y^*)} \left\{ \iint \mu(u(C(Y)) - u(C(Y'))) dF_Y(Y') dF_Y(Y) \right\}
$$

$$
= \frac{\partial}{\partial C(Y^*)} \int \eta \int_{-\infty}^{Y} \{(u(C(Y)) - u(C(Y'))) dF_Y(Y') + \eta \lambda \int_{Y}^{\infty} (u(C(Y)) - u(C(Y'))) dF_Y(Y') \} dF_Y(Y)
$$

$$
= u'(C(Y^*)) (\eta F_Y(Y^*) + \eta \lambda (1 - F_Y(Y^*))) - u'(C(Y^*)) (\eta (1 - F_Y(Y^*)) + \eta \lambda F_Y(Y^*))
$$

$$
= u'(C(Y^*)) \eta (\lambda - 1)(1 - 2F_Y(Y^*)) \text{ with } \eta (\lambda - 1)(1 - 2F_Y(Y^*)) > 0 \text{ for } F_Y(Y^*) < 0.5.
$$

From the above consideration it can be easily inferred that the optimal precommitted portfolio function, if the agent would look up his portfolio his share will be

$$
\alpha_t^c = \frac{\mu - r^f + \frac{\beta \lambda (1 + \frac{\sigma^2}{1 + \gamma^2 (\lambda - 1)(2F_{X(r_1)} - F_{X(r_{t-1})} - 1)} \beta \sigma^2 E[\lambda (\lambda - 1)(\lambda - 2)]}{1 + \gamma^2 (\lambda - 1)(2F_{X(r_1)} - F_{X(r_{t-1})} - 1)} \right)}{\sigma^2}.
$$

Moreover, the optimal precommitted attentive and inattentive consumption shares are

$$
\rho_t^c = \frac{1}{1 + \sum_{\tau=1}^{T-t-1} \beta^\gamma 1 + \gamma (\lambda - 1)(2F_{X(r_1)} - F_{X(r_{t-1})} - 1)} \text{ and } \rho_t'^{cin} = \frac{1}{1 + \sum_{\tau=0}^{T-t-1} \beta^\gamma 1 + \gamma (\lambda - 1)(2F_{X(r_1)} - F_{X(r_{t-1})} - 1)} \left(1 - \sum_{k=1}^{i-1} \rho_t'^{cin} - \sum_{k=1}^{j-1} \rho_t'^{cin} \right).
$$

Thus, the optimal precommitted portfolio share is always lower and the gap increases in good states.

### C.3 Signals about the market

Instantaneous utility is either prospective news utility over the realization of $R$ or prospective news utility over the signal $\tilde{R}$. Prospective news utility over the signal is given by

$$
\gamma \beta \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \mu(u(e^{\log(R)} - y) - u(e^{x-y})) dF_{R^+}(x) dF_y(y),
$$

because the agent separates uncertainty that has been realized in period one, represented by $\tilde{R}$ and $F_{R^+}(x)$, from uncertainty that has not been realized, represented by $F_y(y)$. The agent’s expected news utility from looking up the return conditional on the signal $\tilde{R}$ is given by

$$
\gamma \beta \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \mu(u(e^{\log(\tilde{R})} - y) - u(e^{x-y})) dF_{R^+}(x) dF_y(y).
$$

Thus, the agent expects more favorable news utility over the return when having received a favorable signal. As can be easily seen, expected news utility from checking the return is always less than news utility from knowing merely the signal. The reason is simple, the
agent expects to experience news utility, which is negative on average, over both the signal \( \bar{R} \) and the error \( \epsilon \). Thus, he always prefers to ignore his return when prospective news utility in period one is concerned. But, the difference between the two is smaller when the agent received a more favorable signal. The reason is that the expected news disutility from \( \epsilon \) is less high up on the utility curve, i.e., when \( \bar{R} \) is high.

### C.4 Derivation of the Standard Portfolio-Choice Model

The agent lives for \( t = \{1, ..., T\} \) periods and is endowed with initial wealth \( W_1 \). Each period the agent optimally decides how much to consume \( C_t \) out of his cash-on-hand \( X_t \) and how to invest \( A_t = X_t - C_t \). The agent has access to a risk-free investment with return \( R^f \) and a risky investment with i.i.d. return \( R_t \). The risky investment’s share is denoted by \( \alpha_t \) such that the portfolio return in period \( t \) is given by \( R_t^p = R^f + \alpha_{t-1}(R_t - R^f) \). Additionally, the agent receives labor income in each period \( t \) given by \( Y_t = P_tN_t = P_t e^{it} \) with \( s^T \sim N(0, \sigma^2) \) stochastic up until retirement \( T - R \) and \( P_t \) a deterministic profile. Accordingly, the agent’s maximization problem in each period \( t \) is given by

\[
\max_{C_t} \{ u(C_t) + n(C_t, F_{C_t}^{t-1}) + \gamma \sum_{\tau=1}^{T-t} \beta^\tau n(F_{C_{t+\tau}}^{t-1}) + E_t [\sum_{\tau=1}^{T-t} \beta^\tau U_{t+\tau}] \}
\]

subject to the budget constraint

\[
X_t = (X_{t-1} - C_{t-1})R_t^p + Y_t = A_{t-1}(R^f + \alpha_{t-1}(R_t - R^f)) + P_t e^{it}.
\]

I solve the model by numerical backward induction. The maximization problem in any period \( t \) is characterized by the following first-order condition

\[
u'(C_t) = \frac{\Psi_t' + \gamma \Phi_t'(\eta F_{A_t}^{t-1}(A_t) + \eta \lambda (1 - F_{A_t}^{t-1}(A_t)))}{1 + \eta F_{C_t}^{t-1}(C_t) + \eta \lambda (1 - F_{C_t}^{t-1}(C_t))}
\]

with

\[
\Phi_t' = \beta E_t [R_t^p \frac{\partial C_{t+1}}{\partial X_{t+1}} u'(C_{t+1}) + R_{t+1}^p (1 - \frac{\partial C_{t+1}}{\partial X_{t+1}}) \Phi_{t+1}']
\]

and

\[
\Psi_t' = \beta E_t [R_t^p \frac{\partial C_{t+1}}{\partial X_{t+1}} u'(C_{t+1}) + \eta (\lambda - 1) \int_{C_{t+1}}^\infty (R_{t+1}^p \frac{\partial C_{t+1}}{\partial X_{t+1}} u'(C_{t+1}) - c) dF_{C_{t+1}}^{t+1} \frac{\partial C_{t+1}}{\partial X_{t+1}} c']
\]

\[
+ \gamma \eta (\lambda - 1) \int_{A_{t+1}}^\infty (R_{t+1}^p \frac{\partial A_{t+1}}{\partial X_{t+1}} \Phi_{t+1}' - x) dF_{A_{t+1}}^{t+1} \frac{\partial A_{t+1}}{\partial X_{t+1}} \Phi_{t+1}'(x) + R_{t+1}^p (1 - \frac{\partial C_{t+1}}{\partial X_{t+1}}) \Psi_{t+1}']
\]

Note that, I denote \( \Psi_t = \beta E_t [\sum_{\tau=0}^{T-t} \beta^\tau U_{t+1+\tau}] \), \( \Phi_t = \beta E_t [\sum_{\tau=0}^{T-t} \beta^\tau u(C_{t+1+\tau})] \), \( \Psi_t' = \frac{\partial \Psi_t}{\partial A_{t+1}} \), and \( \Phi_t' = \frac{\partial \Phi_t}{\partial A_{t+1}} \). In turn, the optimal portfolio share can be determined by the following
first-order condition
\[
\gamma \frac{\partial \Phi_t}{\partial \alpha_t} (\eta F_{A_t}^{t-1}(A_t) + \eta \lambda (1 - F_{A_t}^{t-1}(A_t))) + \frac{\partial \Psi_t}{\partial \alpha_t} = 0
\]
or equivalently by maximizing \( \gamma \sum_{\tau = 1}^{T-t} \beta^\tau n(\Phi_{C_t^{t+\tau}}) + \Psi_t \) (maximizing the transformed function \( u^{-1}(\cdot) \) yields more robust results). I use a couple tricks that considerably improve the numerical solution and thus make a structural estimation feasible. In particular, I assume quadrature weights and nodes for the numerical integration, use the endogenous-grid method of Carroll (2001), precompute the marginal value functions in the numerical solution procedures for the optimal consumption and portfolio share function to then use linear interpolation, and transform the marginal value functions \( \frac{\partial \Phi_t}{\partial A_t} \) and \( \frac{\partial \Psi_t}{\partial A_t} \) by \( A_t^\theta \) and \( A_t^{1-\theta} \) respectively to reduce non-linearities. More details on the numerical backward induction solution of a news-utility life-cycle model is provided in Pagel (2013).

D  Proofs

D.1  Proof of Lemma 1

I can rewrite the maximization problem as
\[
r^f + \alpha (\mu - \frac{\sigma^2}{2} - r^f) + \alpha (1 - \alpha) \frac{\sigma^2}{2} + \alpha \sigma E[\eta(\lambda - 1) \int_s^\infty (s - \tilde{s})dF_s(\tilde{s})]
\]
which results in the optimal portfolio share stated in Equation (2) if \( 0 \leq \alpha^* \leq 1 \) and \( \alpha^* = 0 \) if the expression (2) is negative or \( \alpha^* = 1 \) if the expression (2) is larger than one.

D.2  Proof of Proposition 1

The news-utility agent’s optimal portfolio share is
\[
\alpha = \mu - r^f + E[\eta(\lambda - 1) \int_r^\infty (r - \tilde{r})dF_r(\tilde{r})] \sigma^2
\]
It can be easily seen that \( \frac{\partial \alpha}{\partial \eta} < 0 \) and \( \frac{\partial \alpha}{\partial \lambda} < 0 \). As \( E[\eta(\lambda - 1) \int_s^\infty (s - \tilde{s})dF_s(\tilde{s})] < 0 \) even for \( \sigma = 0 \) (note that \( s \sim N(0,1) \)) and the portfolio share is first-order decreasing in \( \sigma \) (the second-order approximation can be found in the text). Now, let me redefine \( \mu \triangleq h\mu \), \( \sigma \triangleq \sqrt{h}\sigma \), and \( r^f \triangleq hr^f \). The optimal portfolio share is given by
\[
\alpha = \frac{\mu - r^f + \sqrt{h} \sigma E[\eta(\lambda - 1) \int_s^\infty (s - \tilde{s})dF_s(\tilde{s})]}{\sigma^2}.
\]
1. Samuelson’s colleague and time diversification: As can be easily seen, as \( \lim \frac{\sqrt{h}}{h} \to \infty \) as \( h \to 0 \), there exists some \( h \) such that \( \mu - r^f < -\frac{\sqrt{h}}{h} \sigma E[\eta(\lambda - 1) \int_s^{\infty} (s - \tilde{s}) dF_\tilde{\lambda}(\tilde{s})] \) and thus \( \alpha = 0 \). As \( \alpha > 0 \) only if

\[
\frac{\mu - r^f}{\sigma} > -E[\eta(\lambda - 1) \int_s^{\infty} (s - \tilde{s}) dF_\tilde{\lambda}(\tilde{s})] \geq 0.
\]

In contrast, \( \alpha^* > 0 \) whenever \( \mu > r^f \). On the other hand, \( \alpha > 0 \) for some \( h \) as if \( h \to \infty \) then \( \lim \frac{\sqrt{h}}{h} \to 0 \) and \( \alpha \to \alpha^* \). Furthermore, it can be easily seen that \( \frac{\partial \alpha}{\partial h} > 0 \).

2. Inattention: Normalized expected utility is \( \frac{EU}{h} = (r^f + \alpha(\mu - \frac{a^2}{2} - r^f) + \alpha(1 - \alpha) \frac{a^2}{2}) + \frac{\sqrt{h}}{h} \sigma E[\eta(\lambda - 1) \int_s^{\infty} (s - \tilde{s}) dF_\tilde{\lambda}(\tilde{s})] \) which is increasing in \( h \) whereas it is constant for the standard agent.

### D.3 Proof of Lemma 2

This proof can be immediately inferred from the derivation of the approximate portfolio share found in Appendix B. The comparative statics in the text hold strictly if \( \alpha^* \in (0, 1) \).

### D.4 Proof of Proposition 2

If the consumption function derived in Appendix C is admissible then the equilibrium exists and is unique as the equilibrium solution is obtained by maximizing the agent’s objective function, which is globally concave, and there is a finite period that uniquely determines the equilibrium. As the consumption share in inattentive periods is constant consumption, \( C_t^{\text{in}} \) is necessarily increasing in \( W_{t-i} - C_{t-i} \) and thus \( r_{t-i} + \ldots + r_{t-i-j_0+1} \) (as \( \frac{\partial \rho_{t-i}}{\partial (r_{t-i} + \ldots + r_{t-i-j_0+1})} < 0 \) which is shown below). For attentive periods, \( \sigma_t^* \) is implicitly defined by the log monotone consumption function restriction \( \frac{\partial \log(C_t)}{\partial (r_{t+i} + \ldots + r_{t-i})} > 0 \) as

\[
\log(C_t) = \log(W_t) + \log(\rho_t)
\]

\[
= \log(W_{t-i} - C_{t-i}) - \sum_{k=1}^{i-1} \frac{C_{t-i+k}^{\text{in}}}{(R^d)^k} \log(R_t^p) + \log(1 + \sum_{T=1}^{T-t} \beta^T \eta^{1+\gamma}(F_{r_T}(r_T) \ldots F_{r_{T-t-i}}(r_{T-i}) + \eta \lambda(1-F_{r_T}(r_T) \ldots F_{r_{T-t-i}}(r_{T-i})))}
\]

as \( \frac{\partial \log(R_t^p)}{\partial (r_{t+i} + \ldots + r_{t-i})} = \frac{\partial \log((R^d)^i + \alpha_{t-i}(R_{t+i} - R_{t-i+i-1}(R^d)^i))}{\partial (R_{t+i} - R_{t-i+i-1})} \frac{\partial (R_{t+i} - R_{t-i+i-1})}{\partial (r_{t+i} + \ldots + r_{t-i+i-1})} \frac{\partial (r_{t+i} + \ldots + r_{t-i+i-1})}{\partial (R_{t+i} - R_{t-i+i-1})} \approx \alpha_{t-i} \frac{\partial R_{t+i} - R_{t-i+i-1}}{\partial (r_{t+i} + \ldots + r_{t-i+i-1})} \) the restrictions are equivalent to \( \frac{\partial \rho_t}{\partial (F_{r_T}(r_T) \ldots F_{r_{T-t-i}}(r_{T-i}))} > -\alpha_{t-i} \frac{\partial (F_{r_T}(r_T) \ldots F_{r_{T-t-i}}(r_{T-i}))}{\partial (r_{t+i} + \ldots + r_{t-i+i-1})} \) then \( \sigma_t^* \) is implicitly defined by the restriction

\[
\frac{\partial \rho_t}{\partial (F_{r_T}(r_T) \ldots F_{r_{T-t-i}}(r_{T-i}))} > -\alpha_{t-i} \frac{\partial (F_{r_T}(r_T) \ldots F_{r_{T-t-i}}(r_{T-i}))}{\partial (r_{t+i} + \ldots + r_{t-i+i-1})}
\]
with
\[
\frac{\partial \rho_t}{\partial (F_t(r_1) \ldots F_t(r_{t+1}))} = -\frac{(1-\gamma)\eta(\lambda-1)\sum_{\tau=1}^{T-t} \beta^\tau \gamma \lambda^{1-}\tau (1-F_t(r_\tau) \ldots F_t(r_{\tau+1}))}{(1+\gamma \lambda (1-F_t(r_\tau) \ldots F_t(r_{\tau+1}))^{T-t}} < 0.
\]

Increasing \( \sigma \) unambiguously decreases \( \frac{\partial (F_t(r_1) \ldots F_t(r_{t+1}))}{\partial (r_{t+1} \ldots + r_{t+1})} > 0 \). Thus, there exists a condition \( \sigma \geq \sigma^*_t \) for all \( t \) which ensures that an admissible consumption function exists. If \( \sigma < \sigma^*_t \) for some \( t \) the agent would optimally choose a flat section that spans the part his consumption function is decreasing. In that situation, the admissible consumption function requirement is weakly satisfied and the model’s equilibrium is not affected qualitatively or quantitatively.

The agent would not choose a suboptimal portfolio share to affect the attentiveness of future selves because the agent is not subject to a time inconsistency with respect to his plan of when to look up his portfolio so long as the agent is not subject to time-inconsistent overconsumption in inattentive periods. The agent does not deviate and overconsume in inattentive periods if the parameter restriction \( \lambda > \frac{1}{\gamma} + \Delta \) holds with \( \Delta \) determined by the following inequalities
\[
\frac{u'(\rho_{i+k}^m)}{1+\eta} < \beta^j u'(\rho_{i+k+j}^m)(1+\eta \lambda) \quad \text{for} \quad k = 1, \ldots, t-2, \quad j = 1, \ldots, i-k-1.
\]
For \( \beta \approx 1 \) and given that \( \rho_{i+k}^m \approx \rho_{i+k+j}^m \), \( \Delta \) is small. In inattentive periods, the agent does not experience news utility in equilibrium because no uncertainty resolves and he cannot fool himself. Therefore, he is not going to deviate from his inattentive consumption path in periods \( t-i+1 \) to \( t-1 \) so long as the bad news from deviating outweigh the good news from doing so in all inattentive periods.

### D.5 Proof of Corollary 1

The proof of Corollary 1 in the dynamic model is very similar to the proof of Proposition 1 in the static model. Please refer to Appendix C for the derivation of the dynamic portfolio share: redefining \( \mu \triangleq h\mu_t, \sigma \triangleq \sqrt{h}\sigma, r^f \triangleq hr^f, \) and \( \beta \triangleq \beta^h, \) the optimal portfolio share (in any period \( t \) as the agent has to look up his portfolio every period) if \( 0 \leq \alpha_t \leq 1 \) can be rewritten as

\[
\alpha_t = \frac{\mu - r^f + \sqrt{h} \sigma}{1+\gamma(\eta \lambda (1-F_t(r_1) \ldots F_t(r_{t+1}))^{T-t}} \sum_{\tau=0}^{T-t-1} \delta^\tau h^{-1} E_t[\eta(\lambda-1) f_s^∞(s-\tilde{s})dF(\tilde{s})]}{\sigma^2}.
\]

- Samuelson’s colleague and time diversification: As can be easily seen, \( \lim \frac{\sqrt{h}}{h} \rightarrow \infty \) as \( h \rightarrow 0 \) whereas \( \lim \frac{1+\gamma(\eta \lambda (1-F_t(r_1) \ldots F_t(r_{t+1}))^{T-t}}{\sum_{\tau=0}^{T-t-1} \delta^\tau h^{-1} E_t[\eta(\lambda-1) f_s^∞(s-\tilde{s})dF(\tilde{s})]} \rightarrow \frac{1+\gamma(\lambda-t-1)}{T-t} \), there exists some \( h \) such that

\[
\mu - r^f > -\frac{\sqrt{h} \sigma}{1+\gamma(\eta \lambda (1-F_t(r_1) \ldots F_t(r_{t+1}))^{T-t}} \sum_{\tau=0}^{T-t-1} \delta^\tau h^{-1} E_t[\eta(\lambda-1) f_s^∞(s-\tilde{s})dF(\tilde{s})]} \quad \text{and thus} \quad \alpha_t = 0 \quad \text{for any} \quad t. \quad \text{In contrast,} \quad \alpha^s > 0 \quad \text{whenever} \quad \mu > r^f. \quad \text{On the other hand,} \quad \alpha_t > 0 \quad \text{for some} \quad h \quad \text{as if} \quad h \rightarrow \infty \quad \text{then} \quad \lim \frac{\sqrt{h}}{h} \rightarrow 0, \quad \lim \frac{1+\gamma(\eta \lambda (1-F_t(r_1) \ldots F_t(r_{t+1}))^{T-t}}{\sum_{\tau=0}^{T-t-1} \delta^\tau h^{-1} E_t[\eta(\lambda-1) f_s^∞(s-\tilde{s})dF(\tilde{s})]} \rightarrow 1, \quad \text{and} \quad \alpha_t \rightarrow \alpha^s. \quad \text{Furthermore, it can be seen that}
\[ \frac{\partial \mu}{\partial h} > 0 \] since

\[ \frac{\partial}{\partial h} \sqrt{h} 1 + \gamma \sum_{t=1}^{T-t-1} \beta^{t-1} h \sum_{r=0}^{T-t-1} \beta^{r-h} = \frac{1}{2\sqrt{h}} h - \sqrt{h} \bigg( \sqrt{h} + \frac{1}{2h} \bigg) \leq 0 \]

\[ + \frac{\sqrt{h} \sum_{k=1}^{T-t-1} \log(\beta) k \beta^{k-h} \sum_{r=0}^{T-t-1} \beta^{r-h} - \frac{\sum_{r=0}^{T-t-1} \log(\beta) \tau \beta^{r-h} (1 + \gamma \sum_{k=1}^{T-t-1} \beta^{k-h})}{(\sum_{r=0}^{T-t-1} \beta^{r-h})^2} < 0 \]

if \( \beta \approx 1 \). The first term is necessarily negative as \( \frac{1}{2\sqrt{h}} h - \sqrt{h} \bigg( \sqrt{h} + \frac{1}{2h} \bigg) < 0 \Rightarrow \frac{1}{2h} h - \sqrt{h} < 0 \Rightarrow \frac{1}{2} h < h \) which is multiplied by \( \frac{1+\gamma \sum_{r=0}^{T-t-1} \beta^{r-h}}{(\sum_{r=0}^{T-t-1} \beta^{r-h})^2} > 0 \). The second term is positive though as \( \sqrt{h} \frac{h}{\sum_{r=0}^{T-t-1} \beta^{r-h}} > 0 \) and the denominator of the fraction is positive while the numerator equals \( (\gamma - 1) \sum_{k=1}^{T-t-1} \log(\beta) k \beta^{k-h} \), which is positive if \( \beta < 1 \). However, if \( \beta \approx 1 \) the first negative term will necessarily dominate the second positive term as its numerator is \( (\gamma - 1) \sum_{t=1}^{T-t-1} \log(\beta) \tau \beta^{r-h} \approx 0 \). Moreover, \( \frac{1+\gamma \sum_{r=0}^{T-t-1} \beta^{r-h}}{(\sum_{r=0}^{T-t-1} \beta^{r-h})^2} \) is decreasing in \( T-t \) if \( \gamma < 1 \) such that \( \alpha_t \) is decreasing in \( T-t \).

- Inattention: Expected utility for any period \( t \), i.e., \( E_{t-1}[U_t] \), is given by

\[ E_{t-1}[U_t] = E_{t-1}[\log(W_t \rho_t) + \alpha_{t-1}(1 + \gamma \sum_{j=1}^{T-t} \beta^{hj}) \sqrt{h} \sigma E[\eta(\lambda - 1) \int_s^\infty (s - \tilde{s})dF(\tilde{s})] \]

If \( \beta \approx 1 \), the change in the \( h \) terms in \( \beta^h \) will be close to zero (as shown in the previous proof), in which case \( \frac{E_{t-1}[U_t]}{h} \) with \( E_{t-1}[\log(W_t \rho_t)] = 0 \) is given by

\[ \frac{E_{t-1}[U_t]}{h} = E_{t-1}[\alpha_{t-1}(1 + \gamma \sum_{j=1}^{T-t} \beta^{hj}) \frac{\sqrt{h}}{h} \sigma E[\eta(\lambda - 1) \int_s^\infty (s - \tilde{s})dF(\tilde{s})]] \]

which is increasing in \( h \) while the standard agent’s normalized expected utility is constant in \( h \).

D.6 Proof of Proposition 3

I pick \( t \) such that \( T-t \) is large and I can simplify the exposition by replacing \( \sum_{k=1}^{T-t} \beta^k \) with \( \frac{\beta}{1-\beta} \). If the agent is attentive every period, his value function is given by

\[ \beta E_{t-1}[V_t(W_t)] = E_{t-1}[\frac{\beta}{1-\beta} \log(W_t) + \psi_t^{t-1}(\alpha_{t-1})] \]

\[ \psi_t^{t-1}(\alpha_{t-1}) = \beta E_{t-1}[\log(\rho_t) + \alpha_{t-1}(1 + \gamma \frac{\beta}{1-\beta}) \sigma E[\eta(\lambda - 1) \int_s^\infty (s - \tilde{s})dF(\tilde{s})]] \]
+ \frac{\beta}{1 - \beta} \log(1 - \rho_t) + \frac{\beta}{1 - \beta} (r^f + \alpha_t(r_{t+1} - r^f) + \alpha_t(1 - \alpha_t)\frac{\sigma^2}{2}) + \psi_{t+1}^t(\alpha_t)].

Now, the agent compares the expected utility from being inattentive to the expected utility from being attentive for one period

\[ E_{t-1}[\log(C_t^{in}) + \frac{\beta}{1 - \beta} \log(W_{t+1}) + \psi_{t+1}^{in}(\alpha_{t-1})] = E_{t-1}[\log(W_{t-1} - C_{t-1}) + \log(R_t) + \log(\hat{\rho}_t^{in})] \]

\[ + \frac{\beta}{1 - \beta} \log(W_{t-1} - C_{t-1} - \frac{C_t^{in}}{R_t}) + \frac{\beta}{1 - \beta} (2r^f + \alpha_{t-1}(r_t + r_{t+1} - 2r^f) + \alpha_{t-1}(1 - \alpha_{t-1})\sigma^2) + \psi_{t+1}^{in}(\alpha_{t-1})] \]

\[ < E_{t-1}[\log(C_t) + \alpha_{t-1}(1 + \gamma \frac{\beta}{1 - \beta})\sigma E[\eta(\lambda - 1) \int_s^\infty (s - \tilde{s})dF(\tilde{s})]] + \frac{\beta}{1 - \beta} \log(W_{t+1}) + \psi_{t+1}^t(\alpha_t)] \]

\[ = E_{t-1}[\log(W_{t-1} - C_{t-1}) + r^f + \alpha_{t-1}(r_t - r^f) + \alpha_{t-1}(1 - \alpha_{t-1})\frac{\sigma^2}{2} + \log(\rho_t) \]

\[ + \alpha_{t-1}(1 + \gamma \frac{\beta}{1 - \beta})\sigma E[\eta(\lambda - 1) \int_s^\infty (s - \tilde{s})dF(\tilde{s})]] + \frac{\beta}{1 - \beta} \log(W_{t+1}) + \psi_{t+1}^t(\alpha_t)] \]

\[ \Rightarrow E_{t-1}[r^d + \log(\hat{\rho}_t^{in}) + \frac{\beta}{1 - \beta} \log(W_{t-1} - C_{t-1}) + \frac{\beta}{1 - \beta} \log(1 - \hat{\rho}_t^{in}) \]

\[ + \frac{\beta}{1 - \beta} (2r^f + \alpha_{t-1}(r_t + r_{t+1} - 2r^f) + \alpha_{t-1}(1 - \alpha_{t-1})\sigma^2) + \psi_{t+1}^{in}(\alpha_{t-1})] \]

\[ < E_{t-1}[r^f + \alpha_{t-1}(r_t - r^f) + \alpha_{t-1}(1 - \alpha_{t-1})\frac{\sigma^2}{2} + \log(\rho_t) + \alpha_{t-1}(1 + \gamma \frac{\beta}{1 - \beta})\sigma E[\eta(\lambda - 1) \int_s^\infty (s - \tilde{s})dF(\tilde{s})] \]

\[ + \frac{\beta}{1 - \beta} \log(W_{t-1} - C_{t-1}) + \frac{\beta}{1 - \beta} (r^f + \alpha_{t-1}(r_t - r^f) + \alpha_{t-1}(1 - \alpha_{t-1})\frac{\sigma^2}{2}) \]

\[ + \frac{\beta}{1 - \beta} \log(1 - \rho_t) + \frac{\beta}{1 - \beta} (r^f + \alpha_{t-1}(r_t + r_{t+1} - r^f) + \alpha_{t-1}(1 - \alpha_{t-1})\frac{\sigma^2}{2}) + \psi_{t+1}^t(\alpha_t)] \]

\[ \Rightarrow E_{t-1}[r^d + \log(\hat{\rho}_t^{in}) + \frac{\beta}{1 - \beta} \log(1 - \hat{\rho}_t^{in}) + \frac{\beta}{1 - \beta} (2r^f + \alpha_{t-1}(r_t + r_{t+1} - 2r^f) + \alpha_{t-1}(1 - \alpha_{t-1})\sigma^2) + \psi_{t+1}^{in}(\alpha_{t-1})] \]

\[ < E_{t-1}[r^f + \alpha_{t-1}(r_t - r^f) + \alpha_{t-1}(1 - \alpha_{t-1})\frac{\sigma^2}{2} + \log(\rho_t) + \alpha_{t-1}(1 + \gamma \frac{\beta}{1 - \beta})\sigma E[\eta(\lambda - 1) \int_s^\infty (s - \tilde{s})dF(\tilde{s})] \]

\[ + \frac{\beta}{1 - \beta} (r^f + \alpha_{t-1}(r_t - r^f) + \alpha_{t-1}(1 - \alpha_{t-1})\frac{\sigma^2}{2}) + \frac{\beta}{1 - \beta} \log(1 - \rho_t) + \frac{\beta}{1 - \beta} (r^f + \alpha_{t-1}(r_t - r^f) + \alpha_{t-1}(1 - \alpha_{t-1})\frac{\sigma^2}{2}) + \psi_{t+1}^t(\alpha_t)] \]

As \( T - t \) is large, for an average period \( t - 1 \) it holds that \( E_{t-1}[\alpha_t] \approx \alpha_{t-1} \) such that

\[ E_{t-1}[r^d + \log(\hat{\rho}_t^{in}) + \frac{\beta}{1 - \beta} \log(1 - \hat{\rho}_t^{in}) + \psi_{t+1}^{in}(\alpha_{t-1})] < E_{t-1}[r^f + \alpha_{t-1}(r_t - r^f) + \alpha_{t-1}(1 - \alpha_{t-1})\frac{\sigma^2}{2}] \]

65
\[ + \log(\rho_t) + \alpha_{t-1}(1 + \gamma \frac{\beta}{1 - \beta}) \sigma E[\eta(\lambda - 1) \int_s^\infty (s - \bar{s})dF(\bar{s})] + \frac{\beta}{1 - \beta} \log(1 - \rho_t) + \psi_{t+1}(\alpha_t) \].

The agent’s continuation utilities are given by

\[ \psi_{t+1}(\alpha_{t-1}) = \beta E_{t-1}[\log(\rho_{t+1}) + \sqrt{2}\alpha_{t-1}(1 + \gamma \frac{\beta}{1 - \beta}) \sigma E[\eta(\lambda - 1) \int_s^\infty (s - \bar{s})dF(\bar{s})]] \]

\[ + \frac{\beta}{1 - \beta} \log(1 - \rho_{t+1}) + \frac{\beta}{1 - \beta}(r^f + \alpha_{t+1}(r_{t+2} - r^f) + \alpha_{t+1}(1 - \alpha_{t+1}) \frac{\sigma^2}{2}) + \psi_{t+2}(\alpha_{t+1}) \]

\[ \psi_t^*(\alpha_t) = \beta E_t[\log(\rho_{t+1}) + \alpha_t(1 + \gamma \frac{\beta}{1 - \beta}) \sigma E[\eta(\lambda - 1) \int_s^\infty (s - \bar{s})dF(\bar{s})]] \]

\[ + \frac{\beta}{1 - \beta} \log(1 - \rho_{t+1}) + \frac{\beta}{1 - \beta}(r^f + \alpha_{t+1}(r_{t+2} - r^f) + \alpha_{t+1}(1 - \alpha_{t+1}) \frac{\sigma^2}{2}) + \psi_{t+2}(\alpha_{t+1})]. \]

The agent’s behavior from period \( t + 2 \) on is not going to be affected by his period \( t \) (in)attentiveness (as his period \( t + 1 \) self can be forced to look up the portfolio by his period \( t \) self). Moreover, as \( T - t \) is large, for an average period \( t - 1 \) it holds that \( E_{t-1}[\alpha_t] \approx \alpha_{t-1} \) such that

\[ E_{t-1}[\psi_{t+1}^*(\alpha_{t-1}) - \psi_t^*(\alpha_t)] = E_{t-1}[(\sqrt{2} - 1)\alpha_{t-1}(1 + \gamma \frac{\beta}{1 - \beta}) \sigma E[\eta(\lambda - 1) \int_s^\infty (s - \bar{s})dF(\bar{s})]] \]

\[ \Rightarrow E_{t-1}[r^d + \log(\rho_{t+1}^\text{in}) + \frac{\beta}{1 - \beta} \log(1 - \rho_{t+1}^\text{in}) + (\sqrt{2} - 2)\alpha_{t-1}(1 + \gamma \frac{\beta}{1 - \beta}) \sigma E[\eta(\lambda - 1) \int_s^\infty (s - \bar{s})dF(\bar{s})]] \]

\[ < E_{t-1}[r^f + \alpha_{t-1}(r_t - r^f) + \alpha_{t-1}(1 - \alpha_{t-1}) \frac{\sigma^2}{2} + \log(\rho_t) + \frac{\beta}{1 - \beta} \log(1 - \rho_t)], \]

which finally results in the following comparison

\[ E_{t-1}[\log(\rho_{t+1}^\text{in}) + \frac{\beta}{1 - \beta} \log(1 - \rho_{t+1}^\text{in})] > E_{t-1}[r^f + \alpha_{t-1}(r_t - r^f) + \alpha_{t-1}(1 - \alpha_{t-1}) \frac{\sigma^2}{2} - r^d] \]

\[ > 0 \text{ in expectation and increasing with } h \]

\[ + \log(\rho_t) + \frac{\beta}{1 - \beta} \log(1 - \rho_t) + (2 - \sqrt{2})\alpha_{t-1}(1 + \gamma \frac{\beta}{1 - \beta}) \sigma E[\eta(\lambda - 1) \int_s^\infty (s - \bar{s})dF(\bar{s})]]. \]

\[ < 0 \text{ and increasing with } \sqrt{h} \]

In turn, if I increase \( h \), I increase the positive return part, which becomes quantitatively relatively more important than the negative news-utility part. Increasing \( h \) implies that the difference between the positive return part and the negative news-utility part will at some point exceed the difference in consumption utilities \( E_{t-1}[\log(\rho_{t+1}^\text{in}) + \frac{\beta}{1 - \beta} \log(1 - \rho_{t+1}^\text{in})] - E_{t-1}[\log(\rho_t) + \frac{\beta}{1 - \beta} \log(1 - \rho_t)] \), which is positive because \( \log(\rho_{t+1}^\text{in}) + \frac{\beta}{1 - \beta} \log(1 - \rho_{t+1}^\text{in}) \) is maximized
for $\rho_t^{in} = \frac{1}{1+\frac{\beta}{1-\beta}}$, which corresponds the standard agent’s portfolio share as the agent is inattentive for just one period. The difference in consumption utilities are given by ($\rho_t = \frac{1}{1+\frac{\beta}{1-\beta} + \frac{1}{\eta}} > \rho_t^{in} = \frac{1}{1+\frac{\beta}{1-\beta}}$ (with $\tilde{\eta} \in (\eta, \eta\lambda)$))

$$\log(\rho_t^{in}) + \frac{\beta}{1-\beta} \log(1-\rho_t^{in}) = \log\left(\frac{1}{1+\beta} + \frac{\beta}{1-\beta} \log\left(\frac{\beta}{1+\beta} \right)ight) = \frac{1}{1-\beta} \log\left(\frac{\beta}{1-\beta} \right) - \left(1 + \frac{\beta}{1-\beta} \right) \log\left(1 + \frac{\beta}{1-\beta} \right)$$

and

$$E_{t-1}[\log(\rho_t) + \frac{\beta}{1-\beta} \log(1-\rho_t)] = E_{t-1}[\log\left(\frac{1}{1+\beta} + \frac{\beta}{1-\beta} \log\left(\frac{\beta}{1+\beta} \right) \right) + \frac{\beta}{1-\beta} \log\left(\frac{\beta}{1+\beta} \right)]$$

$$= E_{t-1}\left[ \frac{\beta}{1-\beta} \log\left(\frac{\beta}{1-\beta} + \frac{1}{\eta} \right) - \left(1 + \frac{\beta}{1-\beta} \right) \log\left(1 + \frac{\beta}{1-\beta} + \frac{1}{\eta} \right) \right].$$

And their difference is given by

$$-(1 + \frac{\beta}{1-\beta}) \log\left(1 + \frac{\beta}{1-\beta}\right) - E_{t-1}\left[ \frac{\beta}{1-\beta} \log\left(\frac{1}{1+\beta} + \frac{1}{\eta} \right) - \left(1 + \frac{\beta}{1-\beta} \right) \log\left(1 + \frac{\beta}{1-\beta} + \frac{1}{\eta} \right) \right],$$

which is decreasing in $\frac{\beta}{1-\beta}$, i.e., $\frac{\partial(-)}{\partial\frac{\beta}{1-\beta}} < 0$, unless $\gamma$ is too small, in which case it is increasing, i.e., $\frac{\partial(-)}{\partial\frac{\beta}{1-\beta}} > 0$, because

$$\frac{\partial(-)}{\partial\frac{\beta}{1-\beta}} = -\log\left(1 + \frac{\beta}{1-\beta}\right) - E_{t-1}\left[ \log\left(\frac{1+\gamma\tilde{\eta}}{1+\gamma\tilde{\eta}} \right) - \log\left(1 + \frac{\beta}{1-\beta} + \frac{1+\gamma\tilde{\eta}}{1+\gamma\tilde{\eta}} \right) \right]$$

and note that

$$\frac{\partial(\log(1 + \frac{\beta}{1-\beta}) - \log(1 + \frac{\beta}{1-\beta} + \frac{1+\gamma\tilde{\eta}}{1+\gamma\tilde{\eta}}))}{\partial\frac{\beta}{1-\beta}} = \frac{1}{1+\frac{\beta}{1-\beta}} - \frac{1}{\frac{1+\tilde{\eta}}{1+\gamma\tilde{\eta}} + \frac{\beta}{1-\beta}} > 0$$

$$\frac{\partial\left(\frac{1+\gamma\tilde{\eta}}{1+\gamma\tilde{\eta}} \right)}{\partial\frac{\beta}{1-\beta}} = \left(1 - \frac{1+\gamma\tilde{\eta}}{1+\gamma\tilde{\eta}} \right)^2 > 0.$$

Thus, an increase in $h$ decreases $\frac{\beta}{1-\beta}$ which increases the difference in consumption utilities (unless $\gamma$ is small such that $\frac{\partial(-)}{\partial\frac{\beta}{1-\beta}} > 0$). If $\frac{\partial(-)}{\partial\frac{\beta}{1-\beta}} < 0$, however, the increase in the difference in consumption utilities due to the increase in $h$ will be less than the rate at which the difference between the return and news-utility part increases if $h$ becomes large. The reason...
is that an increase in \( h \) will result in a decrease in \( \frac{\beta}{1-\beta} \) given by \( \frac{\beta \log(\beta)}{(1-\beta)^2} \), which goes to zero as \( h \to \infty \). Thus, I conclude that the agent will find it optimal to be attentive in every period for \( h > \bar{h} \).

In turn, I can prove that the agent will be inattentive for at least one period if \( h < \bar{h} \). If I decrease \( h \), I decrease the positive return part, which becomes quantitatively less important (as it is proportional to \( h \)) relative to the negative news-utility part (as it is proportional to \( \sqrt{h} \)). Moreover, the difference in consumption utilities speaks towards not looking up the portfolio too, i.e., \( E_{t-1}[\log(\rho_{t}^{in}) + \frac{\beta}{1-\beta} \log(1 - \rho_{t}^{in})] - E_{t-1}[\log(\rho_{t}) + \frac{\beta}{1-\beta} \log(1 - \rho_{t})] > 0 \) as shown above. The intuition for this additional reason to ignore the portfolio is that inattentive consumption is not subject to a self-control problem while attentive consumption is. Furthermore, \( h \) affects the prospective news utility term via \( 1 + \gamma \frac{\beta}{1-\beta} \). However, as \( \frac{\beta}{1-\beta} \) increases if \( h \) decreases this will only make the agent more likely to be inattentive. Thus, I conclude that the agent will find it optimal to be inattentive for at least one period if \( h < \bar{h} \).

It cannot be argued that the agent would behave differently than what is assumed from period \( t+1 \) on unless he finds it optimal to do so from the perspective of period \( t \) because the agent can restrict the funds in the checking account and determine whether or not his period \( t+1 \) self is attentive.

**D.7 Proof of Corollary 2**

Please refer to Appendix C for the derivation of the dynamic portfolio share. The expected benefit of inattention (as defined in the text) is given by

\[
-(\sqrt{i}E[\alpha_{t-1}](1+\gamma \sum_{\tau=1}^{T-1} \beta^\tau) + (\sqrt{j_1}E[\alpha_t] - E[\alpha_{t-1}]\sqrt{j_1} + i)(1+\gamma \sum_{\tau=1}^{T-1-j_1} \beta^\tau))\sigma E[\eta(\lambda-1) \int_s^\infty (s-\bar{s})dF(\bar{s})].
\]

and is always positive if \( E[\alpha_{t-1}] \approx E[\alpha_t] \) (i.e., if \( T-t \) is large). As can be easily inferred that \( \frac{1+\gamma \sum_{\tau=1}^{T-t-j_1} \beta^\tau}{\sum_{\tau=0}^{T-t-j_1} \beta^\tau} \) is converging as \( T-t \) becomes large and thus increases quickly if \( T-t \) is small. Therefore, \( \alpha_t \) is decreasing more quickly, i.e., \( \frac{\partial(\alpha_{t-i} - E_{t-1}[\alpha_t])}{\partial(T-t)} < 0 \), and the expected benefit of inattention is lower if \( E[\alpha_{t-i}] > E[\alpha_t] \) (toward the end of life). The benefit of inattention is lower if \( \alpha_{t-i} > E[\alpha_t] \) and (as I will show in the proof of Proposition 4) \( \frac{\partial \alpha_t}{\partial \gamma} > 0 \). Thus, the benefit of inattention is low if \( r_{t-i} + ... + r_{t-i-j_0+1} \) is low.

**D.8 Proof of Proposition 4**

The agent’s optimal portfolio share is given by

\[
\alpha_t = \mu - r_f + \frac{1+\gamma \sum_{\tau=0}^{T-t-j_1} \beta^\tau E[\eta(\lambda-1) f_{t+1}^{\infty} (r_{t+1-i})dF_i(\bar{r})]}{1+\gamma \eta F_{t}(r_{t+1})...F_i(r_{t-0+1})+\eta \lambda(1-F_{t}(r_{t})...F_i(r_{t-0+1}))}.
\]
As can be easily seen

\[
\frac{\partial \alpha_t}{\partial F_r(r_t) \ldots F_r(r_{t-i+1})} \frac{\partial F_r(r_t) \ldots F_r(r_{t-i+1})}{\partial (r_t + \ldots + r_{t-i+1})}
\]

\[
= \frac{1 + \gamma \sum_{t=0}^{T-t-1} \beta^s}{1 + \gamma \sum_{t=0}^{T-t-1} \beta^s} E[\eta(\lambda-1) \int_{(t-1)\Delta}^{t\Delta} f_{F}(\hat{r})d\hat{r}]
\]

\[
\gamma \eta (\lambda - 1) \frac{\partial F_r(r_t) \ldots F_r(r_{t-i+1})}{\partial (r_t + \ldots + r_{t-i+1})} < 0.
\]

And \(\frac{\sum_{t=0}^{T-t-1} \beta^s}{1 + \gamma \sum_{t=0}^{T-t-1} \beta^s}\) is decreasing in \(T - t\) iff \(\gamma < 1\). And \(\frac{\partial \alpha_t}{\partial r_t - r_{t-i+1}}\) is more negative if \(\frac{\sum_{t=0}^{T-t-1} \beta^s}{1 + \gamma \sum_{t=0}^{T-t-1} \beta^s}\) is high thus the degree of extensive rebalancing is higher late in life.

### D.9 Proof of Corollary 3

The agent’s optimal consumption share is given by

\[
\rho_t = \frac{1}{1 + \sum_{t=1}^{T-t} \beta^s(1 + \gamma \eta F_r(r_t) \ldots F_r(r_{t-i+1}) + \eta \lambda F_r((1 - F_r(r_t)) \ldots F_r(r_{t-i+1})))^{-1}}.
\]

As can be easily seen, \(\frac{1 + \gamma \eta F_r(r_t) \ldots F_r(r_{t-i+1}) + \eta \lambda F_r((1 - F_r(r_t)) \ldots F_r(r_{t-i+1})))}{1 + \gamma \eta F_r(r_t) \ldots F_r(r_{t-i+1}) + \eta \lambda F_r(r_t(r_{t-i+1}))}\) is increasing in \(F_r(r_t) \ldots F_r(r_{t-i+1})\) such that \(\rho_t\) is decreasing in \(F_r(r_t) \ldots F_r(r_{t-i+1})\), i.e.,

\[
\frac{\partial \rho_t}{\partial F_r(r_t) \ldots F_r(r_{t-i+1})} = - \frac{(1 - \gamma) \eta \lambda (\lambda - 1) \sum_{t=1}^{T-t} \beta^s}{(1 + \sum_{t=1}^{T-t} \beta^s(1 + \gamma \eta F_r(r_t) \ldots F_r(r_{t-i+1}) + \eta \lambda F_r((1 - F_r(r_t)) \ldots F_r(r_{t-i+1})))^{-1})^2} < 0 \text{ if } \gamma < 1.
\]

If \(\frac{\partial \rho_t}{\partial F_r(r_t) \ldots F_r(r_{t-i+1})} < 0\), it follows that \(\frac{\partial \rho_t}{\partial R_t - R_{t-i+1}} < 0\) such that if \(\gamma = 0\) (in which case \(\frac{\partial \rho_t}{\partial R_t - R_{t-i+1}} = 0\)) and \(\frac{\partial ((R^f)^i + \alpha_{t-i}(R_{t-1} - R_{t-i+1} - (R^f)^i))(1 - \rho_t)\alpha_t}{\partial R_t - R_{t-i+1}} = \alpha_{t-i}(1 - \rho_t)\alpha_t - (((R^f)^i + \alpha_{t-i}(R_{t-1} - R_{t-i+1} - (R^f)^i))\frac{\partial \rho_t}{\partial R_t - R_{t-i+1}}\alpha_t > \alpha_{t-i}(1 - \rho_t)\alpha_t\). If \(\gamma > 0\), then there exist some threshold \(\tilde{\gamma}\) such that the increasing variation in \(1 - \rho_t\) outweighs the decreasing variation in \(\alpha_t\).

### D.10 Proof of Proposition 5

Comparing the precommitted-monotone and personal-monotone portfolio share it can be easily seen that they are not the same for any period \(t \in \{1, \ldots, T - 1\}\).

1. The precommitted consumption share for attentive consumption is

\[
\rho_t^c = \frac{1}{1 + \sum_{t=1}^{T-t} \beta^s(1 + \gamma \eta (\lambda - 1)(2F_r(r_t) - F_r(r_{t-i+1}) - 1))\eta F_r(r_t) + \eta \lambda (1 - F_r(r_t))) < \eta F_r(r_t) + \eta \lambda (1 - F_r(r_t))\) and the difference increases if \(F_r(r_t) \ldots F_r(r_{t-i+1})\) increases.
2. The precommitted portfolio share is

\[ \alpha_t^c = \mu - rf + \frac{1 + \gamma \sum_{j=1}^{T-t} \beta^T F^\infty_{r} (r_{T-j} - \bar{r}) dF_r(\bar{r})}{1 + \gamma \eta (\lambda - 1) (2F_r(r_t) - F_r(r_{t-1} - 1))} \]

which is lower than the personal-monotone share as \( \eta (\lambda - 1) (2F_r(r_t) - 1) < \eta F_r(r_t) + \eta \lambda (1 - F_r(r_t)) \) and the difference increases if \( F_r(r_t) ... F_r(r_{t-i+1}) \) increases.

3. \( \gamma \) does not necessarily imply an increase in \( \rho_t^c \) because \( \eta (\lambda - 1) (2F_r(r_t) - F_r(r_{t-i+1} - 1) - 1) \) can be negative. Thus, attentive consumption is higher only due to the differences in returns and not due to the time inconsistency any more. Thus, the cost of less consumption utility in period \( t \), is (as defined in the text)

\[ E[\log(\rho_t^{cin})] - E[\log(\rho_t^c)] \]

is lower as \( \rho_t^{cin} = \rho_t^i \) but \( E[\log(\rho_t^i)] < E[\log(\rho_t)] \).

4. As precommitted marginal news utility is always lower and the gap increases in good states there is larger variation in it, i.e., it varies from \( \{-\eta (\lambda - 1), \eta (\lambda - 1)\} \) which is larger than the variation in non-precommitted marginal news \( \{\eta, \eta \lambda\} \), as \( 2\eta (\lambda - 1) > \eta (\lambda - 1) \). Thus, the variation in \( \alpha_t \) (extensive rebalancing) is more pronounced on the pre-committed path. More formally it can be easily seen that

\[ \frac{\partial \alpha_t^c}{\partial F_r(r_t) ... F_r(r_{t-i+1})} < \frac{\partial \alpha_t}{\partial F_r(r_t) ... F_r(r_{t-i+1})} < 0. \]